① $c=1,\ L=\pi,\ f(x)=\sin 4x+\sin 6x-\sin 8x \qquad g(x)=\begin{cases}5 & 0<x<\pi\\ 0 & x=0 \vee x=\pi\end{cases}$ (4p)

$u(x,t)=\sin 4x\cos 4t+\sin 6x\cos 6t-\sin 8x\cos 8t+$ (5p)

$+\frac{20}{\pi}\left(\sin x\sin t+\frac{1}{3^2}\sin 3x\sin 3t+\frac{1}{5^2}\sin 5x\sin 5t+\dots\right)$ (6p)

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① $u(x,t)=\sin 2x\cos 2t-\sin 3x\cos 3t-\sin 5x\cos 5t+$

$+\frac{24}{\pi}\left(\sin x\sin t+\frac{1}{3^2}\sin 3x\sin 3t+\frac{1}{5^2}\sin 5x\sin 5t+\dots\right)$

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② $n=4 \qquad \sum x_i=0 \qquad \sum x_i^2=16+1+4+9=30$ (4p)

$\sum y_i=12 \qquad \sum x_i y_i=-20-4+4-6=-26$ (4p)

$\begin{pmatrix}5 & 0\\ 0 & 30\end{pmatrix}\begin{pmatrix}a\\ b\end{pmatrix}=\begin{pmatrix}12\\ -26\end{pmatrix}\rightarrow a=\frac{12}{5},\ b=\frac{-26}{30}=\frac{-13}{15}$ (6p) (5p)

$y=\frac{12}{5}-\frac{13}{15}x$ (2p)

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② $\begin{pmatrix}5 & 0\\ 0 & 30\end{pmatrix}\begin{pmatrix}a\\ b\end{pmatrix}=\begin{pmatrix}2\\ 42\end{pmatrix} \qquad \frac{2}{5}+\frac{7}{5}x=y$

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③ $\begin{pmatrix}1 & 0\\ 1/\sqrt{2} & 1\end{pmatrix}\begin{pmatrix}1 & 1/\sqrt{2}\\ 0 & 1\end{pmatrix}=\begin{pmatrix}1 & 1/\sqrt{2}\\ 1/\sqrt{2} & 1.5\end{pmatrix}=A^TA$ (4p)

$\det\begin{pmatrix}1-\lambda & 1/\sqrt{2}\\ 1/\sqrt{2} & 1.5-\lambda\end{pmatrix}=(1-\lambda)(1.5-\lambda)-0.5=\lambda^2-2.5\lambda+1$ (4p)

$\lambda_{1/2}=\frac{2.5\pm\sqrt{6.25-4}}{2}=\frac{2.5\pm 1.5}{2}=\begin{cases}2\\ 1/2\end{cases}$ (4p)

$\alpha_1=1/\sqrt{2} \qquad \alpha_2=\sqrt{2}$ (3p)

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③ $A^TA=\begin{pmatrix}\sqrt{2} & 0\\ 1 & \sqrt{2}\end{pmatrix}\begin{pmatrix}\sqrt{2} & 1\\ 0 & \sqrt{2}\end{pmatrix}=\begin{pmatrix}2 & \sqrt{2}\\ \sqrt{2} & 3\end{pmatrix}$

$\lambda^2-5\lambda+4=0 \qquad \lambda_{1/2}=\begin{cases}4\\ 1\end{cases} \qquad \alpha_{1/2}=\begin{cases}2\\ 1\end{cases}$