M1 2.zh 2018.05.15. : B csoport

(1) Kiinduló fv. ffllen rendszer: $1, x, x^2$ 1p, azaz

$$p_0(x) = 1 \quad 1p$$

$$p_1(x) = x - \frac{\langle x,1\rangle}{\langle 1,1\rangle}\cdot 1 = x - \frac{\sum_1^5 x_j}{\sum_1^5 1} 1 = x - \frac{5}{5}1 = x-1 \quad 3p$$

$$p_2(x) = x^2 - \frac{\langle x^2,1\rangle}{\langle 1,1\rangle}1 - \frac{\langle x^2,x-1\rangle}{\langle x-1,x-1\rangle}(x-1) = x^2 - \frac{\sum x_j^2}{5}1 - \frac{\sum x_j^2(x_j-1)}{\sum (x_j-1)^2}(x-1)$$

$$= x^2 - \frac{15}{5}\cdot 1 - 2(x-1) = x^2 - 2\cdot x - 1 \quad 3p$$

$\boxed{\Sigma: 8p}$

(2) Sajátértékek: $\det(A-\lambda I) = (-3-\lambda)(1-\lambda) - (-1)\cdot 4 =$
$= -3 + 2\lambda + \lambda^2 + 4 = \lambda^2 + 2\lambda + 1 = (\lambda+1)^2 = 0 \overset{2p}{\Rightarrow} \lambda_{1,2} = -1.$

Keresendő olyan $h(\lambda,t) = a_0 + a_1\cdot\lambda t$ 1p polinom, melyre

$$\left.\begin{array}{l} h(-1,t) = a_0 - a_1 t = f(-1,t) = e^{-t} \\ h'_\lambda(-1,t) = -a_1 t = f'_\lambda(-1,t) = -te^{-t} \end{array}\right\} 3p$$

Innen $a_1 t = te^{-t}$ 1p és $a_0 = e^{-t} + te^{-t}$ 1p, tehát

$$e^{At} = e^{-t}(1+t)\cdot I + te^{-t}A \quad 2p$$

$\boxed{\Sigma: 10p}$

(3) $y' + 3y = f(t)$, $y(0) = 0$ $\quad /\mathcal{L}$

$$\mathcal{L}(y',s) + 3\underbrace{\mathcal{L}(y,s)}_{Y(s)} = \mathcal{L}(f(t),s)$$

$$s\cdot Y(s) - y(0) + 3\cdot Y(s) = \mathcal{L}(f(t),s) \quad 2p$$

a) $\mathcal{L}(\eta(t),s) = \frac{1}{s}$ 1p $\Rightarrow$ $s\cdot Y(s) + 3Y(s) = \frac{1}{s} \Rightarrow$

$$Y(s) = \frac{1}{s(s+3)} = \frac{\frac{1}{3}}{s} + \frac{-\frac{1}{3}}{s+3} \quad 2p \Rightarrow$$

$$y(t) = \frac{1}{3}\cdot\eta(t) - \frac{1}{3}e^{-3t}\cdot\eta(t) \quad 2p$$

b) $\mathcal{L}(t\cdot\eta(t),s) = (-1)\cdot(\mathcal{L}(\eta(t),s))' = (-1)\cdot\left(\frac{1}{s}\right)' =$
$= (-1)\cdot\frac{-1}{s^2} = \frac{1}{s^2}$ 2p

$$Y(s) = \frac{1}{s^2}\cdot\frac{1}{s+3} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+3} = \frac{A(s+3)s + B(s+3) + Cs^2}{s^2(s+3)}$$

s^2: $A + C = 0$ $\qquad C = \frac{1}{9}$
s: $3A + B = 0 \Rightarrow A = -\frac{1}{9}$
k: $3B = 1 \Rightarrow B = \frac{1}{3}$

$$\Rightarrow Y(s) = -\frac{1}{9}\cdot\frac{1}{s} + \frac{1}{3}\cdot\frac{1}{s^2} + \frac{1}{9}\cdot\frac{1}{s+3} \quad 2p \Rightarrow$$

$$y(t) = -\frac{1}{9}\eta(t) + \frac{1}{3}\cdot t\cdot\eta(t) + \frac{1}{9}\cdot e^{-3t}\cdot\eta(t) \quad 2p$$

c) $\mathcal{L}(\eta(t+4),s) = \frac{e^{4s}}{s}$ 1p

$$Y(s) = \frac{e^{4s}}{s(s+3)} = e^{4s}\left(\frac{1/3}{s} - \frac{1/3}{s+3}\right) \quad 1p \Rightarrow$$

$$y(t) = \frac{1}{3}\cdot\eta(t+4) - \frac{1}{3}\cdot e^{-3(t+4)}\cdot\eta(t+4) \quad 2p$$

$\boxed{\Sigma: 17p}$

(4) 1 íven a hibák száma (ξ) Poisson: $P(\xi = k) = e^{-\lambda}\cdot\frac{\lambda^k}{k!}$ 2p

Tudjuk, hogy $P(\xi = 0) = e^{-\lambda} = \frac{1}{\sqrt[5]{e}} \Rightarrow \lambda = \frac{1}{5}$ 2p

1 íven a hibák száma ≤ 2: 2p
$$P(\xi=0) + P(\xi=1) + P(\xi=2) = \frac{1}{\sqrt[5]{e}} + \frac{1}{5}\cdot\frac{1}{\sqrt[5]{e}} + \frac{1}{2}\cdot\frac{1}{25}\cdot\frac{1}{\sqrt[5]{e}} = (*)$$

Hibák száma >2: $P(\xi > 2) = 1 - (*) =: p$. 2p
(1 íven belül)

τ: azon ívek száma, amelyen a hiba > 2 $\sim$ binom(10, p) 3p

$$P(\tau \leq 2) = P(\tau=0) + P(\tau=1) + P(\tau=2) \quad 2p =$$

$$= \sum_{k=0}^{2}\binom{10}{k}p^k(1-p)^{10-k} = \ldots \quad 2p$$

$\boxed{\Sigma: 15p}$