M1 2.zh. 2018.05.15: A csoport

① Kiinduló fgv-ek rendre: $1, x, x^2$, 1p azaz

$p_0(x) = 1$ 1p

$p_1(x) = x - \frac{\langle x,1\rangle}{\langle 1,1\rangle}1 = x - \frac{\sum x_i}{\sum 1}1 = x - \frac{-5}{5}1 = x+1$ 3p

$p_2(x) = x - \frac{\langle x^2,1\rangle}{\langle 1,1\rangle}1 - \frac{\langle x^2,x+1\rangle}{\langle x+1,x+1\rangle}(x+1) = x^2 - \frac{\sum x_i^2}{\sum 1}1 - \frac{\sum x_i^2(x_i+1)}{\sum(x_i+1)^2}(x+1) =$

$= x^2 - \frac{15}{5}1 - (-2)(x+1) = x^2 + 2x - 1$ 3p

$\Sigma: 8p$

② Sajátértékek: $\det(A-\lambda I) = (2-\lambda)(-2-\lambda) - 3\cdot(-1) =$

$= -4 + 2\lambda - 2\lambda + \lambda^2 + 3 = \lambda^2 - 1 = (\lambda-1)(\lambda+1) = 0$ 2p $\lambda_{1,2} = \pm 1$

Keresendő olyan $h(\lambda,t) = a_0 + a_1\cdot\lambda t$ 1p, melyre

$$\left.\begin{aligned} h(-1,t) &= a_0 - a_1 t = f(-1,t) = e^{-t} \\ h(1,t) &= a_0 + a_1 t = f(1,t) = e^{t} \end{aligned}\right\} 3p$$

Innen $a_0 = (e^t + e^{-t})\frac{1}{2}$ 1p és $a_1 t = \frac{1}{2}(e^t - e^{-t})$ 1p, tehát

$e^{At} = h(A,t) = \frac{1}{2}(e^t + e^{-t})I + \frac{1}{2}(e^t - e^{-t})A$ 2p $\Sigma: 10p$

③ $y' - 4y = f(t)$, $y(0) = 0$ $/\mathcal{L}$

$\mathcal{L}(y',s) - 4\cdot\mathcal{L}(y,s) = s\cdot Y(s) - y(0) - 4\cdot Y(s) \overset{2p}{=} \mathcal{L}(f,s)$

a, $\mathcal{L}(\eta(t),s) = \frac{1}{s}$ 1p $\Rightarrow$ $Y(s) = \frac{1}{s(s-4)} = \frac{-\frac{1}{4}}{s} + \frac{\frac{1}{4}}{s-4}$ 2p

$y(t) = -\frac{1}{4}\cdot\eta(t) + \frac{1}{4}\cdot e^{4t}\cdot\eta(t)$ 2p

b, $\mathcal{L}(t\cdot\eta(t),s) = (-1)\cdot(\mathcal{L}(\eta(t),s))' = \frac{1}{s^2}$ 2p

$Y(s) = \frac{1}{s^2(s-4)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s-4} = \frac{As(s-4) + B(s-4) + Cs^2}{s^2(s-4)}$

s^2: $A + C = 0$ $\Rightarrow C = \frac{1}{16}$

s: $-4A + B = 0 \Rightarrow A = -\frac{1}{16}$

konst: $-4B = 1 \Rightarrow B = -\frac{1}{4}$

$\Rightarrow Y(s) = -\frac{1}{16}\cdot\frac{1}{s} - \frac{1}{4}\cdot\frac{1}{s^2} + \frac{1}{16}\cdot\frac{1}{s-4}$ 2p

$y(t) = -\frac{1}{16}\cdot\eta(t) - \frac{1}{4}\cdot t\eta(t) + \frac{1}{16}\cdot e^{4t}\cdot\eta(t)$ 2p

c, $\mathcal{L}(\eta(t-3),s) = \frac{e^{-3s}}{s}$ 1p

$Y(s) = e^{-3s}\cdot\frac{1}{s(s-4)} = e^{-3s}\cdot\left(\frac{-1/4}{s} + \frac{1/4}{s-4}\right)$ 1p

$y(t) = -\frac{1}{4}\cdot\eta(t-3) + \frac{1}{4}e^{4(t-3)}\eta(t-3)$ 2p

$\Sigma: 17p$

④ ξ: 1 gépre érkező vírusok száma $\sim$ Poisson, azaz

$P(\xi = k) = \frac{\lambda^k}{k!}e^{-\lambda}$ 2p $\Rightarrow P(\xi = 0) = \frac{1}{e^3} \Rightarrow \lambda = 3$ 2p

1 gépen 1-nél több vírus:

$P(\xi \geq 2) = 1 - P(\xi < 2) = 1 - P(\xi = 0) - P(\xi = 1) \overset{2p}{=}$

$= 1 - \frac{1}{e^3} - \frac{3}{e^3} = 1 - \frac{4}{e^3} =: p$ 2p

τ: azon gépek száma, melyekre legalább 2 vírus érkezett $\sim$ binom$(20,p)$ 3p

$P(\tau \leq 2) = P(\tau = 0) + P(\tau = 1) + P(\tau = 2) \overset{2p}{=}$

$= \sum_{k=0}^{2}\binom{20}{k}\cdot p^k(1-p)^{20-k} \overset{2p}{=} \ldots$

$\Sigma: 15p$