M1 2018 1 zh. megoldások (B)

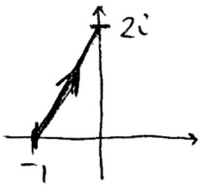
(1) $v(x,y)=\alpha(x^2y+xy)-4y^3-3 \Rightarrow \Delta v=0$ kell ahhoz, hogy képzetes rész legyen:

$$\left.\begin{aligned} v'_x(x,y)&=\alpha(2xy+y) \Rightarrow v''_{xx}(x,y)=2\alpha y\\ v'_y(x,y)&=\alpha(x^2+x)-12y^2 \Rightarrow v''_{yy}(x,y)=-24y \end{aligned}\right\} \Delta v=2\alpha y-24y=0 \quad \alpha=12$$

Ekkor $f'(z)=v'_y(x,y)+i\cdot v'_x(x,y)=12(x^2+x)-12y^2+i\cdot 12(2xy+y)$, $z=1+2i$

tehát $f'(1+2i)=24-48+i\cdot 12(4+2)=-24+i\cdot 72$

(2) a) $\displaystyle\int_{\gamma_1}\frac{\bar z\cdot \mathrm{Im}\, z}{\mathrm{Re}(z)+1}\,dz$,



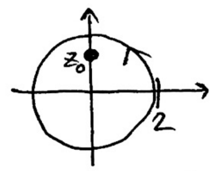
$\gamma_1: g(t)=t+i(2t+2)$, $t\in[-1,0]$

$$\| $$

$$\int_{-1}^{0}\frac{(t-i(2t+2))\cdot(2t+2)}{t+1}\cdot(1+2i)\,dt=(1+2i)\cdot\Big[t^2-2i(t^2+2t)\Big]_{-1}^{0}=$$

$$=(1+2i)\Big(0-(1-2i(1-2))\Big)=-(1+2i)^2=3-4i$$

b) $\displaystyle\int_{\gamma_2}\frac{e^{\sin(iz)}}{(z-i\pi)^2}+\underbrace{z^2\,\mathrm{ch}\, z}\,dz$, $\gamma_2: |z|=4$:



szing. hely: $z_0=i\pi$ $\in \mathrm{int}\,\gamma_2$

reguláris a tartományon $\Rightarrow \int=0$ a Cauchy-tétel alapján

|| Cauchy-∫form.

$$2i\pi\cdot\left[\frac{d}{dz}\left(e^{\sin(iz)}\right)\right]_{z=i\pi}=2i\pi\cdot\Big[e^{\overset{0}{\widetilde{\sin(iz)}}}\cdot\underbrace{\cos(iz)}_{-1}\cdot i\Big]_{z=i\pi}=2\pi.$$

(3)

$$f(z)=\frac{2z-3i}{(z-i)(z-2i)}=\frac{1}{z-i}+\frac{1}{z+2i}=\frac{i}{1-\frac{z}{i}}+\frac{\frac12 i}{1-\frac{z}{2i}}=$$

$$=\frac{i}{\frac{z}{i}}\cdot\frac{1}{\frac{i}{z}-1}+\frac{\frac12 i}{1-\frac{z}{2i}}=\frac{1}{z}\cdot\frac{1}{1-\frac{i}{z}}+\frac{1/2i}{1-\frac{z}{2i}}=\frac{1}{z}\cdot\sum_0^\infty\left(\frac{i}{z}\right)^k+\frac12 i\cdot\sum_0^\infty\left(\frac{z}{2i}\right)^k$$

$1<|z|<2$

(4)

$$A=\begin{bmatrix}1&-3&9\\1&-2&4\\1&-1&1\\1&0&0\\1&1&1\end{bmatrix}\quad x=\begin{bmatrix}a\\b\\c\end{bmatrix}\quad f=\begin{bmatrix}1\\3\\6\\1\\0\end{bmatrix}$$

$$A^TAx=A^Tf$$

$$\begin{bmatrix}5&-5&15\\-5&15&-35\\15&-35&99\end{bmatrix}\begin{bmatrix}a\\b\\c\end{bmatrix}=\begin{bmatrix}11\\-15\\27\end{bmatrix}$$

$c=-1$, $b=-\frac{12}{5}$, $a=\frac{14}{5}$

$\Rightarrow p(t)=a+bt+ct^2=\dots$.