M1 2018 1. zh megoldások (A)

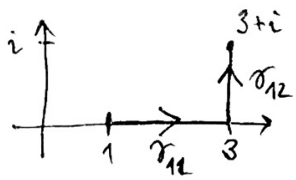
(1) $v(x,y) = cx^2 + 2xy - 2y^2 \Rightarrow \Delta v = 0$ kell ahhoz, hogy képzetes rész lehessen:

$$v'_x(x,y) = 2cx + 2y \Rightarrow v''_{xx}(x,y) = 2c$$
$$v'_y(x,y) = 2x - 4y \Rightarrow v''_{yy}(x,y) = -4$$
$$\Delta v = 2c - 4 = 0, \quad c = 2$$

Ekkor $f'(z) = v'_y(x,y) + i v'_x(x,y) = 2x - 4y + i(4x+2y)$, $\quad -i = 0 - 1\cdot i$

tehát $f'(-i) = 4 + i(-2) = 4 - 2i$

(2) a) $\displaystyle\int_{\gamma_1} e^{2\bar z}\,dz$,



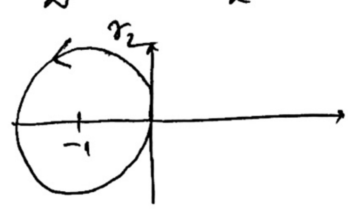
$\gamma_{11}: g_1(t) = t,\ t\in[1,3]$

$\gamma_{12}: g_2(t) = 3 + it,\ t \in [0,1]$

$$\int_{\gamma_1} e^{2\bar z}\,dz = \int_{\gamma_{11}} + \int_{\gamma_{12}} = \int_1^3 e^{2t}\cdot 1\,dt + \int_0^1 e^{2(3-it)}\cdot i\,dt =$$

$$= \left[\frac{e^{2t}}{2}\right]_1^3 + i\left[\frac{e^{2(3-it)}}{-2i}\right]_0^1 = \frac{1}{2}e^6 - \frac{1}{2}e^2 - \frac{1}{2}e^{2(3-i)} + \frac{1}{2}e^6$$

b) $\displaystyle\int_{\gamma_2} \frac{z\cdot\cos\left(\frac{\pi}{4}(z-1)^2\right)}{z^2-1}\,dz$, $\quad \gamma_2: |z+1| = 1$



$z^2 - 1 = (z-1)(z+1)$

$z_1 = 1 \notin \text{int}\,\gamma_2$

$z_2 = -1 \in \text{int}\,\gamma_2$

$$\Rightarrow \int_{\gamma_2} \frac{\frac{z\cos\left(\frac{\pi}{4}(z-1)^2\right)}{z-1}}{z+1}\,dz \overset{\text{Cauchy-}\int\text{form.}}{=} 2\pi i\cdot\frac{(-1)\cdot\cos\left(\frac{\pi}{4}\cdot(-1-1)^2\right)}{-1-1} = -i\pi$$

(3) $$f(z) = \frac{z-1+i}{(z-i)^2\cdot(z-(1+i))} = \frac{1}{(z-i)^2}\cdot\frac{z-(1+i)+2i}{z-(1+i)} = \frac{1}{(z-i)^2}\left(1 + \frac{2i}{z-(1+i)}\right) =$$

$z_0 = i$

$$= \frac{1}{(z-i)^2}\left(1 + \frac{2i}{(z-i)-1}\right) = \frac{1}{(z-i)^2}\left(1 + \frac{2i}{z-i}\cdot\frac{1}{1-\frac{1}{z-i}}\right) =$$

$\left|\frac{1}{z-i}\right| < 1 \Leftrightarrow |z-i| > 1$

$$= \frac{1}{(z-i)^2}\left(1 + \frac{2i}{z-i}\cdot\sum_0^\infty \frac{1}{(z-i)^k}\right)$$

(4) $$A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{bmatrix} \quad x = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \quad f = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \\ 0 \end{bmatrix}$$

$A^TAx = A^Tf$ $$\begin{bmatrix} 5 & 5 & 15 \\ 5 & 15 & 35 \\ 15 & 35 & 99 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}$$

$c = \frac{15}{70}$, $b = -\frac{37}{70}$, $a = \frac{3}{35}$

$\Rightarrow p(t) = a + bt + ct^2 = \dots$