

06 - Laplace transforms, exercises

1.

Find the Laplace transforms of the following functions.

- a) $7 \sin 3t$ b) $6t^2 + 3t - 2$ c) $t \cos 7t$
d) $e^{2t} \sin 3t$ e)* $t e^{-t} \cos 4t$ f)* $t^2 \sin 5t$

2.

Find the inverse Laplace transform.

- a) $\frac{3}{s} + \frac{1}{s-5} - \frac{7}{s-2}$ b) $\frac{11}{s-3} + \frac{4}{s^2-25}$ c) $\frac{7}{s^2+4}$ d) $\frac{s+4}{s^2+9}$
e) $\frac{3}{s^2+4s+14}$ f) $\frac{4}{s^2+2s}$ g) $\frac{3}{s^3+2s^2}$

3.

Solve the following initial value problems by the Laplace transform.

- a) $y' = y, y(0) = 3$
b) $y' = 7y, y(0) = -1$
c) $y'' = -y, y(0) = 0, y'(0) = -2$
d) $y'' = -y, y(0) = 1, y'(0) = 0$
e) $2y' - y = 0, y(0) = \frac{1}{2}$
f) $y' + 7y = 6, y(0) = 0$
g) $2y' + y = e^{2t}, y(0) = 1$
h) $y'' + 3y' + 2y = e^{-t}, y(0) = 0, y'(0) = 0$
i) $y'' + 2y' + 5y = 0, y(0) = 1, y'(0) = 0$
j) $y' + y = \sin 3t, y(0) = 0$

4.*

Solve the following initial value problems.

- a) $x' = 2x + y, y' = -y, x(0) = 0, y(0) = -3$
b) $x' = x + 3y, y' = -x + 5y, x(0) = 1, y(0) = 0$
c) $x' = -8y, y' = 2x, x(0) = 1, y(0) = -2$
d) $x' = 3x - 2y, y' = 2x + 5y, x(0) = 1, y(0) = 1$
e) $x' = 3x, y' = x + 3y, x(0) = 4, y(0) = 2$
f) $x' = 3x + y, y' = -x + y, x(0) = 4, y(0) = 2$
g) $x' = 4y + 1, y' = x + t, x(0) = 1, y(0) = 0$
h) $x' = x + 3y + 8, y' = x - y, x(0) = 0, y(0) = 0$