
04 - Higher order linear differential equations, exercises

Homogeneous equations

1. Solve the following homogeneous linear differential equations:

a) $y'' - 8y' + 15y = 0$	b) $y'' + 2y' = 0$
c) $y'' - 8y' + 16y = 0$	d) $y'' + 4y' + 13y = 0$
e) $y'' + 25y = 0$	f) $y''' + 2y'' + y' = 0$
g) $y^{(4)} - y = 0$	h) $y^{(4)} - y''' = 0$

2. Solve the following initial value problems:

a) $y'' + 2y' + 2y = 0, y(0) = 2, y'(0) = 1$
b) $y'' + 3y' - 4y = 0, y(0) = 3, y'(0) = -4$
c) $y'' + 10y' + 25y = 0, y(0) = -1, y'(0) = 7$

Nonhomogeneous equations

3. Solve the following nonhomogeneous differential equations:

a) $y'' - 5y' + 6y = 2 \sin 2t$	b) $y'' - 5y' + 6y = 2t e^t$
c) $y'' - 6y' + 13y = 39$	d) $y'' - y' - 2y = 3e^{2t}$
e) $y'' - 3y' + 2y = e^{3t} + 4t^2 - 6$	f) $y'' - 3y' + 2y = t + e^t$
g) $y'' - 2y' + y = 6e^t$	h) $y'' + 8y' + 25y = e^{-4t}$
i) $y'' + 2y' = 2t + 3$	j) $y'' + y = \sin t$

4. The current I in an RLC circuit is described by the equation

$$L I''(t) + R I'(t) + \frac{1}{C} I(t) = F(t). \text{ Let } L = 1, R = 3, C = 0.5.$$

Find the form of the particular solution if

a) $F(t) = e^t$	b) $F(t) = e^{-t}$
c) $F(t) = 2t + 1$	d) $F(t) = t$
e) $F(t) = t^2$	f) $F(t) = 3$
g) $F(t) = \sin 2t$	h) $F(t) = e^{-t} \sin 2t$
i) $F(t) = e^{2t} - \cos t$	j) $F(t) = 4 + e^{-t}$

Homework

5. Find the general solution of the equation $L I''(t) + R I'(t) + \frac{1}{C} I(t) = 0$ if

a) $R = 0, L > 0, C > 0$

b) $R = 0, L = C = 2$

c) $L = 1, R = 3, C = 0.4$

d) $L = 2, R = 10, C = 0.08$

e) $L = 2, R = 4.5, C = 1$

6. Solve the following initial value problems:

a) $y'' + 16y = 0, y(0) = 3, y'(0) = -2$

b) $y'' - 2y' + y = e^{3t}, y(0) = 1, y'(0) = 1$

7. Find the general solution of the equation $y''(t) + 4y(t) = f(t)$ where

a) $f(t) = 2t$

b) $f(t) = 4t^2$

c) $f(t) = \cos 2t$

8. Find the general solution of the equation

$$L I''(t) + R I(t) + \frac{1}{C} I(t) = F_0 e^{\gamma t} \text{ where } R, L, C > 0.$$