# Practice exercises 6.

1. Find the local extrema of the following functions:

a) $f(x, y) = 2x^3 - 6x + 5 + y^3 - 12y$

b) $f(x, y) = (x - 3y + 3)^2 + (x - y - 1)^2$

c) $f(x, y) = (x - y + 1)^2 - \left(x^2 - 2\right)^2$

2. Let $f(x, y) = x^3 y^5$

a) Find the absolute (global) extrema of $f$ on the following set:

$A = \left\{(x, y) \in \mathbb{R}^2 :\ 0 \le x \le 1,\ 0 \le y \le x\right\}$

b) Does $f$ have a local extremum if $(x, y) \in \mathbb{R}^2$?

3. Least squares method:

Some physical quantity $y$ is assumed to be directly proportional to another quantity $x$, that is, $y = \mathrm{A}\mathrm{x} + b$. The two quantities are measured $n$ times, that is, we have the measurement data $(x_1, y_1),\ (x_2, y_2),\ \ldots,\ (x_n, y_n)$. How should we choose $A$ and $B$ so that the straight line $y = Ax + B$ is closest to the measured points, by which we mean that the expression is minimal?

$$f(A, B) = \sum_{k=1}^{n} (y_k - (A x_k + B))^2$$

Homework:

4. Find the local extrema of the following functions:

a) $f(x, y) = 2x + y + \dfrac{4}{xy}$

b) $f(x, y) = x^4 + y^4 - x^2 - 2xy - y^2$

5. Find the absolute (global) extrema of $f$ on the given set:

a) $f(x, y) = x^2 + y^2 - 2x - 2y$, $\quad H = \left\{(x, y) \in \mathbb{R}^2 : x \ge 0,\ y \ge 0,\ y \le 9 - x\right\}$

b) $f(x, y) = x^2 + y^2 - xy - 3x$, $\quad H = \left\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \le 3\right\}$

c) $f(x, y) = \dfrac{xy}{x^2 + y^2}$, $\quad H = \left\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \le 1\right\}$

d) $f(x, y) = x^3 + y^3 - 9xy + 27$, $H = \left\{(x, y) \in \mathbb{R}^2 : 0 \le x \le 4,\ 0 \le y \le 4\right\}$