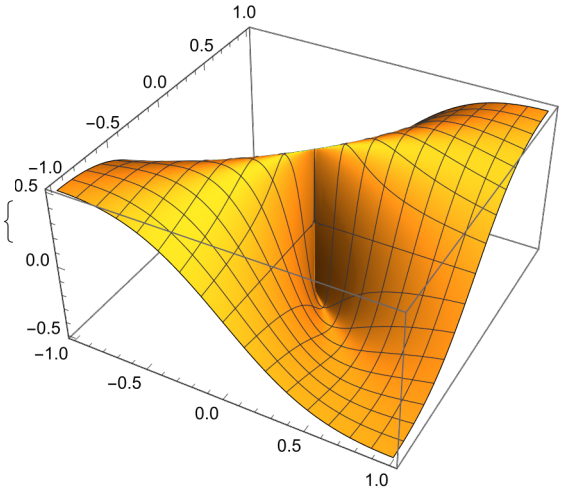
Some two-variable functions and their graphs

1. $f(x, y) = \frac{x\,y}{x^2+y^2}$

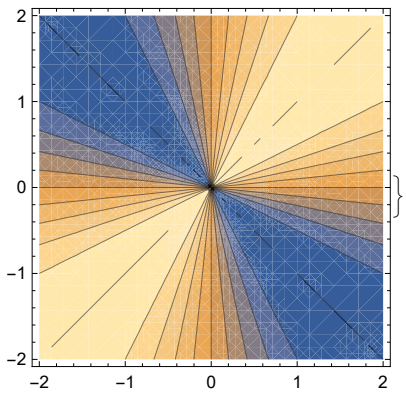
https://www.wolframalpha.com/input?i=plot+%28x+y%29%2F%28x%5E2+%2B+y%5E2%29

In[1]:=
```
f[x_, y_] := x y / (x^2 + y^2);
{Plot3D[f[x, y], {x, -1, 1}, {y, -1, 1},
  PlotPoints → 300, BoxRatios → Automatic, ImageSize → 300],
 ContourPlot[f[x, y], {x, -2, 2}, {y, -2, 2}, BoxRatios → Automatic, ImageSize → 200]}
```

Out[2]=



,

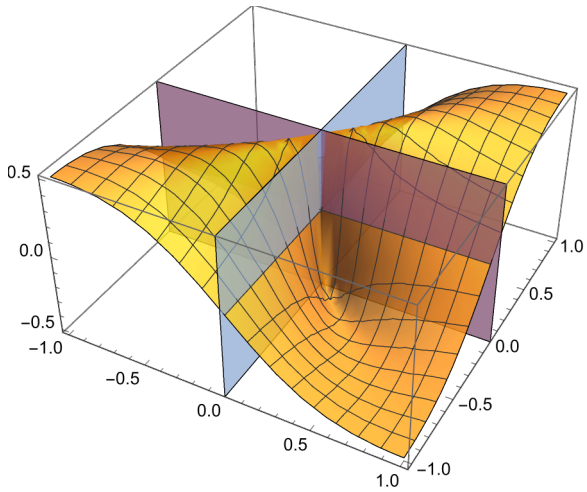


}

Intersecting the graph of f with vertical planes:

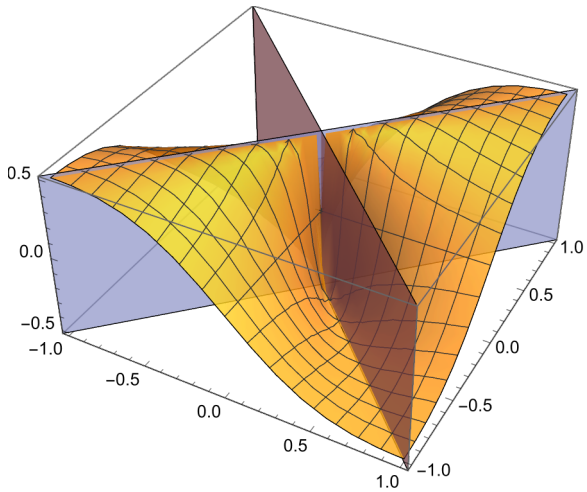
In[4]:=
```
Show[Plot3D[x y/(x^2 + y^2), {x, -1, 1}, {y, -1, 1},
  PlotStyle → Opacity[0.8], BoxRatios → Automatic, ImageSize → 300],
 Graphics3D[{Opacity[0.7], InfinitePlane[{{0, 0, 0}, {0, 0, 1}, {0, 1, 0}}]}],
 Graphics3D[{Opacity[0.7], InfinitePlane[{{0, 0, 0}, {0, 0, 1}, {1, 0, 0}}]}]]
```

Out[4]=



In[5]:=
```
Show[Plot3D[x y/(x^2 + y^2), {x, -1, 1}, {y, -1, 1},
  PlotStyle → Opacity[0.8], BoxRatios → Automatic, ImageSize → 300],
 Graphics3D[{Opacity[0.7], InfinitePlane[{{0, 0, 0}, {1, 1, 0}, {0, 0, 2}}]}],
 Graphics3D[{Opacity[0.7], InfinitePlane[{{0, 0, 0}, {1, -1, 0}, {0, 0, 2}}]}]]
```

Out[5]=

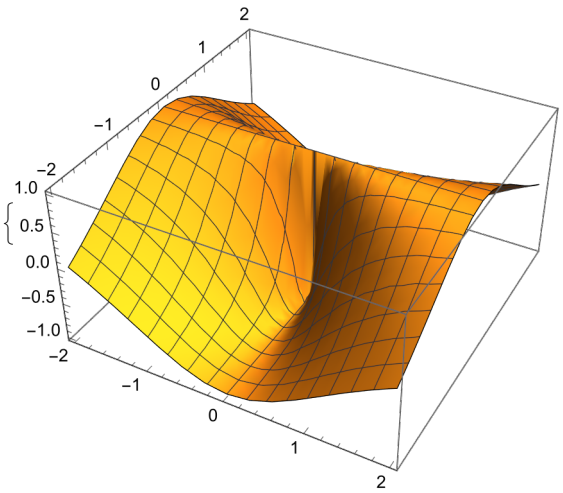


2. $f(x, y) = \frac{x^2-y^2}{x^2+y^2}$

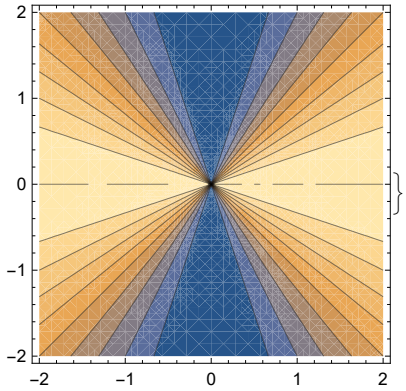
https://www.wolframalpha.com/input?i=plot+%28x%5E2+-+y%5E2%29%2F%28x%5E2+%2B+y%5E2%29

In[6]:=
```
f[x_, y_] := (x^2 - y^2)/(x^2 + y^2);
{Plot3D[f[x, y], {x, -2, 2}, {y, -2, 2}, BoxRatios → Automatic, ImageSize → 300],
 ContourPlot[f[x, y], {x, -2, 2}, {y, -2, 2}, BoxRatios → Automatic, ImageSize → 200]}
```

Out[7]=



,

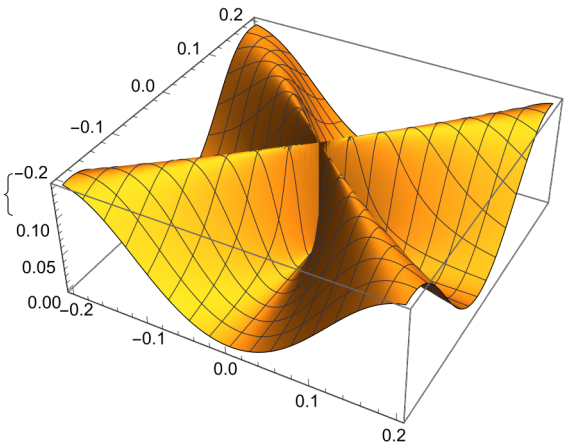


3. $f(x, y) = \frac{x^2 y^2}{3x^4+4y^4}$

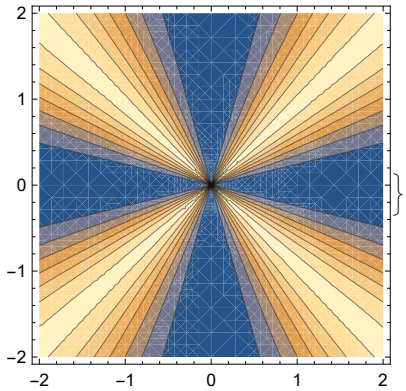
https://www.wolframalpha.com/input?i=plot+%28x%5E2+y%5E2%29%2F%283x%5E4%2B4y%5E4%29

In[10]:=
```
f[x_, y_] := (x^2 y^2)/(3 x^4 + 4 y^4);
{Plot3D[f[x, y], {x, -0.2, 0.2}, {y, -0.2, 0.2},
  PlotPoints → 100, BoxRatios → Automatic, ImageSize → 300],
 ContourPlot[f[x, y], {x, -2, 2}, {y, -2, 2}, BoxRatios → Automatic, ImageSize → 200]}
```

Out[11]=



,

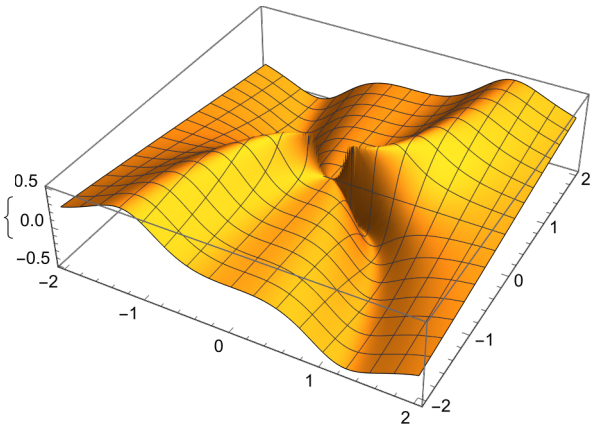


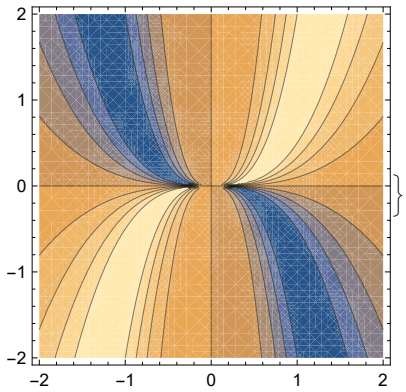
4. $f(x, y) = \frac{x^3 y}{x^6+y^2}$

https://www.wolframalpha.com/input?i=plot+%28x%5E3+y%29%2F%28x%5E6%2By%5E2%29

In[12]:=
```
f[x_, y_] := x^3 y / (x^6 + y^2);
{Plot3D[f[x, y], {x, -2, 2}, {y, -2, 2},
  PlotPoints → 100, BoxRatios → Automatic, ImageSize → 300],
 ContourPlot[f[x, y], {x, -2, 2}, {y, -2, 2}, BoxRatios → Automatic, ImageSize → 200]}
```

Out[13]=

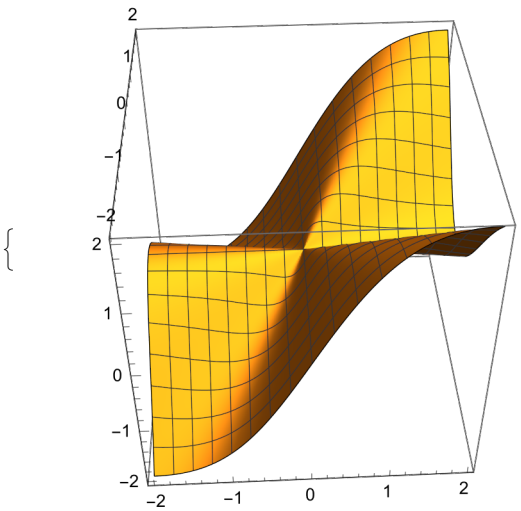


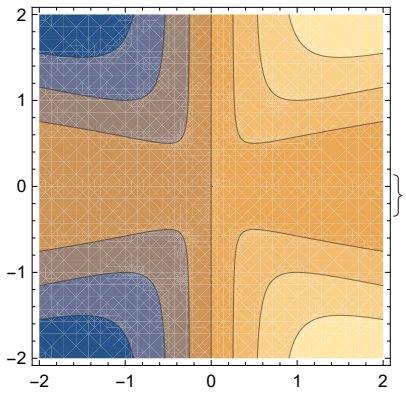


5. $f(x, y) = \frac{2 x y^2}{x^2+y^2}$

In[14]:=
```
f[x_, y_] := 2 x y^2 / (x^2 + y^2);
{Plot3D[f[x, y], {x, -2, 2}, {y, -2, 2},
  PlotPoints → 100, BoxRatios → Automatic, ImageSize → 300],
 ContourPlot[f[x, y], {x, -2, 2}, {y, -2, 2}, BoxRatios → Automatic, ImageSize → 200]}
```

Out[15]=

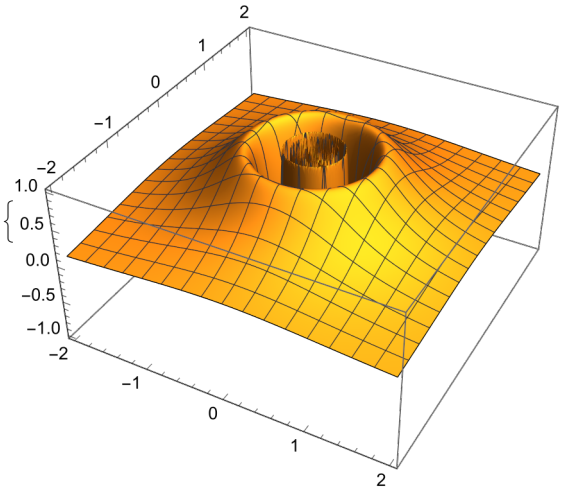


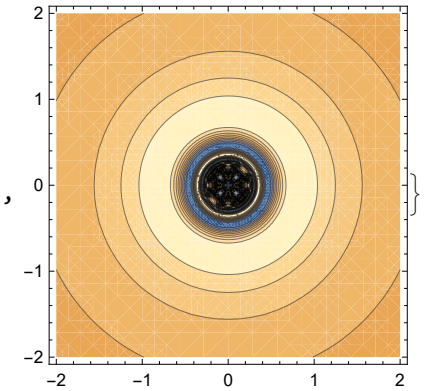


6. $f(x, y) = \sin\left(\frac{1}{x^2+y^2}\right)$

In[16]:=
```
f[x_, y_] := Sin[1/(x^2 + y^2)];
{Plot3D[f[x, y], {x, -2, 2}, {y, -2, 2},
  PlotPoints → 200, BoxRatios → Automatic, ImageSize → 300],
 ContourPlot[f[x, y], {x, -2, 2}, {y, -2, 2}, BoxRatios → Automatic, ImageSize → 200]}
```

Out[17]=





7. $f(x, y) = x\,y\left(x^2 + y^2 - 1\right)$

In[18]:=
```
f[x_, y_] := x y (x^2 + y^2 - 1);
{Plot3D[f[x, y], {x, -1.2, 1.2}, {y, -1.2, 1.2}, BoxRatios → Automatic, ImageSize → 300],
 ContourPlot[f[x, y], {x, -2, 2}, {y, -2, 2}, BoxRatios → Automatic, ImageSize → 200]}
```

Out[19]=