

Practice exercises 3.

1. Find the following limits if they exist:

$$a) \lim_{(x,y) \rightarrow (0,0)} \frac{xy + 3}{x^2y + 4}$$

$$b) \lim_{(x,y) \rightarrow (0,0)} \frac{\sin(x^2y)}{x^2 \cos(y^2)}$$

$$c) \lim_{(x,y) \rightarrow (0,0)} \arctan(xy) \cdot \sin \frac{1}{x^2 + y^2}$$

$$d) \lim_{(x,y) \rightarrow (0,0)} \frac{xy}{2x^2 + 2y^2}$$

$$e) \lim_{(x,y) \rightarrow (0,0)} \frac{3xy^3}{2x^2 + 2y^2}$$

$$f) \lim_{(x,y) \rightarrow (0,0)} \frac{xy^2}{2x^2 + 3y^2}$$

$$g) \lim_{(x,y) \rightarrow (0,0)} \frac{3x^2 + 5y^2}{2x^2 + y^2}$$

$$g) \lim_{(x,y) \rightarrow (2,1)} \frac{e^{x^2-3y}}{1+2x^2+3y^2}$$

$$h) \lim_{(x,y) \rightarrow (1,1)} \frac{xy - 1}{x - 1}$$

$$i) \lim_{(x,y) \rightarrow (0,0)} \frac{x^2 + y}{x - y}$$

$$j) \lim_{(x,y) \rightarrow (0,0)} \frac{\sin(xy)}{\sqrt{x^2 + y^2}}$$

$$k) \lim_{(x,y) \rightarrow (0,0)} \frac{\ln(1 + x^2y)}{x^2 + y^2}$$

2. Investigate the continuity of the following functions:

$$a) f(x, y) = \begin{cases} \frac{3x^2y^2}{4x^4 + 7y^4}, & \text{if } (x, y) \neq (0, 0) \\ 0, & \text{if } (x, y) = (0, 0) \end{cases} \quad b) f(x, y) = \begin{cases} (2x + 3y) \ln(x^2 + y^2), & \text{if } (x, y) \neq (0, 0) \\ 0, & \text{if } (x, y) = (0, 0) \end{cases}$$

$$b) f(x, y) = \begin{cases} \frac{x^3 - y^3}{xy}, & \text{if } xy \neq 0 \\ 0, & \text{if } xy = 0 \end{cases}$$

3. Show an example for a function $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ such that

- (a) for all $x_0 \in \mathbb{R}$ the function $f(x_0, \cdot) : \mathbb{R} \rightarrow \mathbb{R}$, $y \mapsto f(x_0, y)$ is continuous,
- (b) for all $y_0 \in \mathbb{R}$ the function $f(\cdot, y_0) : \mathbb{R} \rightarrow \mathbb{R}$, $x \mapsto f(x, y_0)$ is continuous,
however, f is not continuous.

4. Consider the function $f(x, y) = \frac{xy^2}{x^2 + y^4}$ if $(x, y) \neq (0, 0)$ and $f(0, 0) = 0$.

- (a) Is the function continuous along the straight lines passing through the origin?
- (b) Is the function continuous at the origin?