

Practice exercises 1.

1. Calculate the following improper integrals.

$$a) \int_0^\infty x e^{-x^2} dx$$

$$b) \int_0^\infty \frac{1}{1+x} dx$$

$$c) \int_{-\infty}^\infty \frac{1}{2+3x^2} dx$$

$$d) \int_0^2 \frac{1}{\sqrt{4-x^2}} dx$$

$$e) \int_{-\frac{1}{2}}^1 \frac{1}{\sqrt{2x+1}} dx$$

$$f) \int_2^\infty \frac{1}{x \ln x} dx$$

$$g) \int_{-1}^1 \frac{1}{x^2} dx$$

$$h) \int_0^\infty e^{-x} \cos x dx$$

$$i) \int_0^1 \ln^n x dx \quad (n \in \mathbb{N})$$

$$j) \int_0^\infty e^{-\sqrt{x}} dx$$

$$k) \int_3^\infty \frac{1}{x^2 - 3x + 2} dx$$

$$l) \int_0^2 \frac{1}{x^2 - 4x + 3} dx$$

$$m) \int_0^\infty \frac{\sqrt{x}}{(1+x)^2} dx \quad (\text{substitution: } t = \sqrt{x})$$

$$n) \int_0^{\frac{\pi}{2}} \frac{1}{(\sin x + \cos x)^2} dx \quad (\text{substitution: } t = \tan x)$$

2. Decide whether the following integrals converge or diverge.

$$a) \int_3^\infty \frac{x^{\frac{3}{2}} + 1 - \sqrt{x}}{x^2 + 1} dx$$

$$b) \int_1^\infty \frac{e^{-x}}{x} dx$$

$$c) \int_0^1 \frac{1}{\sqrt{1-x^3}} dx$$

$$d) \int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{\sin x}} dx$$

$$e) \int_a^b \frac{1}{(x-a)(x-b)} dx$$

$$f) \int_0^\infty \frac{x^2 \cos^2(x^5 + 3)}{x^4 + 3x^2 + 5} dx$$

$$3. \text{ Let } f : [0, \infty) \rightarrow \mathbb{R}, \quad f(x) = \begin{cases} \frac{\sin x}{x}, & \text{if } x \neq 0 \\ 1, & \text{if } x = 0 \end{cases}$$

Show that the improper integral $\int_0^\infty f$ is convergent, but $\int_0^\infty |f|$ is divergent.

4. Decide whether the following series converge or diverge.

$$a) \sum_{n=2}^\infty \frac{1}{n(\ln n)^{\alpha+1}} \quad (\alpha \geq 0)$$

$$b) \sum_{n=2}^\infty \frac{1}{n(\ln n)(\ln \ln n)^{\alpha+1}} \quad (\alpha \geq 0)$$

$$c) \sum_{n=1}^\infty \frac{1}{e^{\sqrt{n}}}$$

$$d) \sum_{n=0}^\infty \frac{\operatorname{sh} n}{1 + \operatorname{ch}^2 n}$$

$$e) \sum_{n=1}^\infty \frac{\cos(\frac{1}{n})}{n^2 \sin(\frac{1}{n})}$$

$$f) \sum_{n=1}^\infty \frac{n}{(n^2 + 1)^{\frac{3}{2}}}$$