Practice exercises 14.

1. Find the values of the following definite integrals.

a) $\int_0^{\pi} (1+\cos x)\,\mathrm{d}x$ b) $\int_0^1 x\,e^{-3x}\,\mathrm{d}x$ c) $\int_{-1}^1 \sqrt{1-x^2}\,\mathrm{d}x$ d) $\int_0^{\ln 2} \sqrt{e^x-1}\,\mathrm{d}x$

2. Compute the area of the region enclosed by the following curves.

a) $f(x)=x^2+2x,\ \ g(x)=4-x^2$ b) $y=\ln x,\ \ y=0,\ \ x=\frac{1}{e},\ \ x=e$

3. Calculate the derivatives of the following functions.

a) $A(x)=\int_0^x \frac{1}{\sqrt{1+t^4}}\,\mathrm{d}t$ b) $B(x)=\int_0^{x^3} \frac{1}{\sqrt{1+t^4}}\,\mathrm{d}t$ c) $C(x)=\int_x^{x^3} \frac{1}{\sqrt{1+t^4}}\,\mathrm{d}t$

4. Calculate the following limits: a) $\lim\limits_{x\to 0} \frac{\int_0^x \ln(1+t)\,\mathrm{d}t}{x^2}$ b) $\lim\limits_{x\to 0} \frac{\int_0^{x^2} \sqrt{1+t^4}\,\mathrm{d}t}{x^2}$

5. Calculate the following improper integrals. In cases *i*) and *j*) substitute $t=\sqrt{x}$.

a) $\int_0^{\infty} x\,e^{-x^2}\,\mathrm{d}x$ b) $\int_0^{\infty} \frac{1}{1+x}\,\mathrm{d}x$ c) $\int_{-\infty}^{\infty} \frac{1}{2+3x^2}\,\mathrm{d}x$ d) $\int_0^2 \frac{1}{\sqrt{4-x^2}}\,\mathrm{d}x$

e) $\int_{-\frac{1}{2}}^1 \frac{1}{\sqrt{2x+1}}\,\mathrm{d}x$ f) $\int_2^{\infty} \frac{1}{x\ln x}\,\mathrm{d}x$ g) $\int_{-1}^1 \frac{1}{x^2}\,\mathrm{d}x$ h) $\int_0^{\infty} e^{-x}\cos x\,\mathrm{d}x$

i) $\int_0^{\infty} e^{-\sqrt{x}}\,\mathrm{d}x$ j) $\int_0^{\infty} \frac{\sqrt{x}}{(1+x)^2}\,\mathrm{d}x$ j) $\int_5^{\infty} \frac{1}{x^2-4x+3}\,\mathrm{d}x$ k) $\int_0^2 \frac{1}{x^2-4x+3}\,\mathrm{d}x$

6. Decide whether the following integrals converge or diverge.

a) $\int_3^{\infty} \frac{x^{\frac{3}{2}}+1-\sqrt{x}}{x^2+1}\,\mathrm{d}x$ b) $\int_1^{\infty} \frac{e^{-x}}{x}\,\mathrm{d}x$

7. Prove that $\int_1^{\infty} \frac{1}{x(\ln x)^{\alpha}}\,\mathrm{d}x$ is finite if and only if $\alpha>1$.

8. Find the arc length of the following curves on the given intervals:

a) $f(x)=x^2,\ \ x\in[0,1]$ b) $f(x)=\cosh x,\ \ x\in[-\ln 2,\ln 2]$

9. Calculate the volume of the following bodies of rotation (the graph of f is rotated about the x axis over the given interval).

a) $f(x)=\sqrt{x},\ \ x\in[0,4]$ b) $f(x)=e^x,\ \ x\in[0,2]$

c) $f(x)=\sqrt{\cos x},\ \ x\in\left[0,\frac{\pi}{2}\right]$ d) $f(x)=\frac{1}{\cos x},\ \ x\in\left[0,\frac{\pi}{4}\right]$

10. Calculate the surface area of the following bodies of rotation (the graph of f is rotated about the x axis over the given interval).

a) $f(x)=x^3,\ \ x\in[0,1]$ b) $f(x)=\sqrt{x+1},\ \ x\in[0,2]$