Practice exercises 12.

Taylor polynomial, Taylor series

1. Find the following Taylor polynomials of order n and center x_0.

a) $f(x) = \sin x,\ \ x_0 = \dfrac{\pi}{3},\ n = 3$ b) $f(x) = 2^x,\ \ x_0 = 1,\ n = 3$

c) $f(x) = \tan x,\ \ x_0 = 0,\ n = 3$ d) $f(x) = \tan x,\ \ x_0 = \dfrac{\pi}{4},\ n = 2$

e) $f(x) = \log(1 - x),\ \ x_0 = 1,\ n = 4$ f) $f(x) = \arccos x^2,\ \ x_0 = 0,\ n = 2$

2. Let $P(x)$ be a polynomial of degree n. Prove that the Taylor polynomial of order n corresponding to $P(x)$ at any center $x_0 \in \mathbb{R}$ is $P(x)$ itself.

3. Estimate the value of $\sqrt{65}$ by the Taylor polynomial of order 2 of $f(x) = \sqrt{x}$ at center 64. Give an upper bound for the error of the approximation.

4. Estimate the value of $\log 1.2$ by the Taylor polynomial of order 3 of $f(x) = \log(1 + x)$ at center 0. Give an upper bound for the error of the approximation.

5. Estimate the value of $\sinh 1$ by an appropriate Taylor polynomial with error less than 10^{-2}.

6. Calculate the Taylor series of the following function with center x_0 and find the radius of convergence.

a) $f(x) = \sin 2x,\ \ x_0 = \pi$ b) $f(x) = 3^x,\ \ x_0 = 1$

c) $f(x) = \dfrac{1}{x - 2},\ \ x_0 = 0$ d) $f(x) = \dfrac{1}{x - 2},\ \ x_0 = 5$

e) $f(x) = \dfrac{1}{x^2 + 3},\ \ x_0 = 0$ f) $f(x) = \dfrac{x^5}{x^2 + 3},\ \ x_0 = 0$

g) $f(x) = \dfrac{1}{(1 - x)^2},\ x_0 = 2$ h) $f(x) = \sinh 3x^3,\ \ x_0 = 0$

i) $f(x) = \arccos x,\ \ x_0 = 0$ j) $f(x) = \cosh x,\ \ x_0 = -1$

k) $f(x) = x^2 \cdot \sqrt[3]{64 - 8x^2},\ \ x_0 = 0$ l) $f(x) = \dfrac{x}{\sqrt{x - 1}},\ \ x_0 = 3$