Calculus 1, Sample Midterm Test 2

1. Estimate the value of $\cosh 0.5$ by the appropriate Taylor polynomial with error less than 10^{-3}. (9 points)

2. Calculate the Taylor series of the function $f(x) = x^2 e^x$, with center $x_0 = -2$, and find the radius of convergence. (9 points)

3. Analyze the function $f(x) = xe^{-x^2}$ and plot its graph. (18 points)

4. Find the following limit: $\lim_{x\to\pi/2} \sin x^{1/\cos x}$ (9 points)

5. You want to make a rectangular tin cup with square base (open top!) of volume 1 liter. What is the minimal possible surface area of the cup? (12 points)

6. The equation $x^2y^4 + xy^3 = 12$ describes a curve on the plane. Find the derivative $\frac{dy}{dx}$ and the second derivative $\frac{d^2y}{dx^2}$ at the point $(3, 1)$ of the curve. (12 points)

7. Prove that $|\tan x - \tan y| \geq |x - y|$. (7 points)

8. Evaluate the following integrals: (8+8+8 points)

   a; $\int x^2 \ln x dx$

   b; $\int \sin^3 2x \cos 2x dx$

   c; $\int \frac{1}{x^3 - x} dx$