Calculus 1, Sample Midterm Test 1

1. Let $a_0 = 6$ and $a_{n+1} = 7 - \frac{10}{a_n}$. Prove by induction that $a_n \geq 5$ for every n, and that the sequence a_n is monotonically decreasing. (8 points)

2. Let $a_n = \frac{n\sqrt[3]{n}-2}{n^3}$. Find the limit of a_n and provide a threshold index N for $\varepsilon = 0.01$ (you don't need to find the smallest possible threshold index, just find any). (7 points)

3. Find the limit of the following sequences: (7+7+7+7 points)

   $a_n = \sqrt[3]{1-n^3} + \sqrt[3]{n^3+2n}$.

   $b_n = \sqrt[n]{\frac{5^n+3n+n^2}{n+2}}$

   $c_n = \left(\frac{n^2+5n-6}{n^2-2n-2}\right)^{3n+2}$

   $d_n = \frac{\sin(n!)}{10^n+n^{10}}$

4. Decide whether the following series converge or not: (7+7+7 points)

   $a)\ \sum_{n=1}^{\infty} \frac{(-1)^n}{n+\sqrt[n]{n}}$

   $b)\ \sum_{n=1}^{\infty} \frac{10^n n^2}{n!}$

   $c)\ \sum_{n=1}^{\infty} \frac{n!(2n)!}{(3n)!}$

5. For what values of $x \in \mathbb{R}$ does the following series converge: (9 points)

   $\sum_{n=1}^{\infty} \frac{4^{n+1}}{n+3}(x+2)^n$

6. Decide whether the following set is open, closed or neither: (6 points)

   $\mathbb{Q} \setminus (0,1)$

7. Let $f(x) = \dfrac{\sin(x^2+x+a)}{x+3}$ if $x \neq -3$, and $f(-3) = b$. Choose the parameters a and b so that the function f be continuous at $x_0 = -3$. (7 points)

8. Differentiate the following functions: (7+7 points)

   a; $f(x) = \operatorname{tg}(3x^2+1)\sin(1/\sqrt[3]{x})$

   b; $g(x) = \cos\left(\frac{x^2+3x}{\sqrt{x^4+4}}\right)$