Practice exercises 13.

1. Find the values of the following definite integrals.

a) $\int_0^{\pi} \cos^2 x \, \mathrm{dx}$ b) $\int_0^1 x\, e^{-3x} \, \mathrm{dx}$ c) $\int_0^2 e^{|2x-1|} \, \mathrm{dx}$

2. Compute the area of the region enclosed by the following curves.

a) $f(x) = x^2 + 2x, \;\; g(x) = 4 - x^2$ b) $y = \ln x, \;\; y = 0, \;\; x = \frac{1}{e}, \;\; x = e$

3. Calculate the derivatives of the following functions.

a) $A(x) = \int_0^x \frac{1}{\sqrt{1+t^4}} \, \mathrm{dt}$ b) $B(x) = \int_0^{x^3} \frac{1}{\sqrt{1+t^4}} \, \mathrm{dt}$ c) $C(x) = \int_x^{x^3} \frac{1}{\sqrt{1+t^4}} \, \mathrm{dt}$

4. Calculate the following limits: a) $\lim\limits_{x \to 0} \frac{\int_0^x \ln(1+t) \, \mathrm{dt}}{x^2}$ b) $\lim\limits_{x \to 0} \frac{\int_0^{x^2} \sqrt{1+t^4} \, \mathrm{dt}}{x^2}$

5.* Find the arc length of the following curves on the given intervals:

a) $f(x) = x^2, \;\; x \in [0, 1]$ b) $f(x) = \cosh x, \;\; x \in [-\ln 2, \ln 2]$

6. Calculate the volume of the following bodies of rotation (the graph of f is rotated about the x axis over the given interval).

a) $f(x) = \sqrt{x}, \;\; x \in [0, 4]$ b) $f(x) = e^x, \;\; x \in [0, 2]$

c) $f(x) = \sqrt{\cos x}, \;\; x \in \left[0, \frac{\pi}{2}\right]$ d) $f(x) = \frac{1}{\cos x}, \;\; x \in \left[0, \frac{\pi}{4}\right]$

7. Calculate the surface area of the following bodies of rotation (the graph of f is rotated about the x axis over the given interval).

a) $f(x) = x^3, \;\; x \in [0, 1]$ b) $f(x) = \sqrt{x+1}, \;\; x \in [0, 2]$

Results

1. Find the values of the following definite integrals.

a) $\int_0^{\pi} \cos^2 x \, \mathrm{dx}$ b) $\int_0^1 x\, e^{-3x} \, \mathrm{dx}$ c) $\int_0^2 e^{|2x-1|} \, \mathrm{dx}$

https://math.bme.hu/~tasnadi/merninf_anal_1/anal1_gyak.pdf

1. a) page 96, exercise 11.

c) page 97, exercise 13.

b)

https://www.wolframalpha.com/input?i=integrate+xe%5E%28-3x%29

https://www.wolframalpha.com/input?i=integrate+xe%5E%28-3x%29%2C+x%3D0+to+1

2. Compute the area of the region enclosed by the following curves.

a) $f(x) = x^2 + 2x, \;\; g(x) = 4 - x^2$ b) $y = \ln x, \;\; y = 0, \;\; x = \frac{1}{e}, \;\; x = e$

The area between the graphs of $f(x)$ and $g(x)$ if $x \in [a, b]$ is:

$$A = \int_a^b |\, f(x) - g(x) \,|\, \mathrm{dx}$$

https://math.bme.hu/~tasnadi/merninf_anal_1/anal1_gyak.pdf

2. a) page 98, exercise 15., see figure 5.1 on page 99.

b) page 99, exercise 16., see figure 5.2 on page 100.

3. Calculate the derivatives of the following functions.

a) $A(x) = \int_0^x \frac{1}{\sqrt{1+t^4}}\,\mathrm{dt}$ b) $B(x) = \int_0^{x^3} \frac{1}{\sqrt{1+t^4}}\,\mathrm{dt}$ c) $C(x) = \int_x^{x^3} \frac{1}{\sqrt{1+t^4}}\,\mathrm{dt}$

https://math.bme.hu/~tasnadi/merninf_anal_1/anal1_gyak.pdf

3. page 104, exercise 21.

4. Calculate the following limits: a) $\lim_{x \to 0} \frac{\int_0^x \ln(1+t)\,\mathrm{dt}}{x^2}$ b) $\lim_{x \to 0} \frac{\int_0^{x^2} \sqrt{1+t^4}\,\mathrm{dt}}{x^2}$

Apply the L’Hospital’s rule.

a) $\frac{1}{2}$ b) 1

5.* Find the arc length of the following curves on the given intervals:

a) $f(x) = x^2, \;\; x \in [0, 1]$ b) $f(x) = \cosh x, \;\; x \in [-\ln 2, \ln 2]$

The arc length can be calculated as $L = \int_a^b \sqrt{1 + (f'(x))^2}\,\mathrm{dx}$

a) $\frac{1}{4}\left(2\sqrt{5} + \operatorname{arsinh}(2)\right)$ b) $\frac{3}{2}$

6. Calculate the volume of the following bodies of rotation (the graph of f is rotated about the x axis over the given interval).

a) $f(x) = \sqrt{x}, \;\; x \in [0, 4]$ b) $f(x) = e^x, \;\; x \in [0, 2]$

c) $f(x) = \sqrt{\cos x}, \;\; x \in \left[0, \frac{\pi}{2}\right]$ d) $f(x) = \frac{1}{\cos x}, \;\; x \in \left[0, \frac{\pi}{4}\right]$

The volume can be calculated as $V = \pi \int_a^b f^2(x)\,\mathrm{dx}$

a) $f^2(x) = x \implies \pi \int x\,\mathrm{dx} = \frac{\pi x^2}{2} \qquad \implies V = \pi \int_a^b x\,\mathrm{dx} = 8\pi$

b) $f^2(x) = e^{2x} \implies \pi \int e^{2x}\,\mathrm{dx} = \frac{\pi}{2} \cdot e^{2x} \qquad \implies V = \pi \int_0^2 e^{2x}\,\mathrm{dx} = \frac{\pi}{2} \cdot (e^4 - 1)$

c) $f^2(x) = \cos x \implies \pi \int \cos x\,\mathrm{dx} = \pi \sin x \qquad \implies V = \pi \int_0^{\pi/2} \cos x\,\mathrm{dx} = \pi$

d) $f^2(x) = \frac{1}{\cos^2 x} \implies \pi \int \frac{1}{\cos^2 x} \, \mathrm{dx} = \pi \tan x \implies V = \pi \int_0^{\pi/4} \frac{1}{\cos^2 x} \, \mathrm{dx} = \pi$

7. Calculate the surface area of the following bodies of rotation (the graph of f is rotated about the x axis over the given interval).

a) $f(x) = x^3, \; x \in [0, 1]$ b) $f(x) = \sqrt{x+1}, \; x \in [0, 2]$

The volume can be calculated as $A = 2\pi \int_a^b f(x) \sqrt{1 + (f'(x))^2} \, \mathrm{dx}$

a) $f(x) \sqrt{1 + (f'(x))^2} = x^3 \sqrt{1 + 9x^4}$

$\implies \int x^3 \sqrt{1 + 9x^4} \, \mathrm{dx} = \frac{1}{54} \left(1 + 9x^4\right)^{3/2}$

$\implies A = \frac{\pi}{27} \left(10\sqrt{10} - 1\right)$

b) $f(x) \sqrt{1 + (f'(x))^2} = \sqrt{x+1} \sqrt{1 + \frac{1}{4(x+1)}} = \frac{1}{2} \sqrt{4x+5}$

$\implies \int \frac{1}{2} \sqrt{4x+5} \, \mathrm{dx} = \frac{1}{12} (5 + 4x)^{3/2}$

$\implies A = \pi \left(\frac{9}{2} - \frac{5\sqrt{5}}{6} \right)$

Additional exercises: from page 86:
https://math.bme.hu/~tasnadi/merninf_anal_1/anal1_gyak.pdf