Syllabus - Calculus 1 (BMETE92AM36)

1. Set theoretical basis

   Logical symbols, truth tables, negation of statements, proof by contradiction, set theoretical operations

2. Real numbers, complex numbers

   Basic arithmetical operations, ordering, fractional parts, Bernoulli inequality, binomial theorem, absolute value, triangle inequality, mathematical induction, arithmetic of complex numbers, arithmetic-geometric mean inequality

3. Topology of the real line

   Open sets, closed sets, bounded sets, interior, exterior, boundary, closure of a set, dense sets, compact sets, Cantor intersection theorem, Borel-Lebesgue theorem (possibly without proof)

4. Sequences

   The notion of limit. Monotone sequences, subsequences, accumulation points, Bolzano-Weierstrass theorems. Liminf, limsup. Cauchy criterion. Limit of specific well-known sequences.

5. Numeric series

   Convergence of a series, partial sums, Cauchy criterion. Majorant criterion, ratio criterion, root criterion. Leibniz-type series. Absolute and conditional convergence. Cauchy product. Mertens theorem, Abel rearrangement. Elementary functions (exp, log, sin, cos, sh, ch) and their identities.

6. Real functions

   Notion of even, odd monotone, periodic functions. Convex, concave functions, Jensen-inequality. Limits, one-sided limits, continuity, transference principle. Properties of continuous functions: topological characterization, Bolzano theorem. Continuous image of compact set is compact, Weierstrass min-max principle, uniform continuity, Heine theorem.

7. Differentiation

   Notion of the derivative, its relation to continuity. Derivative of sums, products, quotients, chain rule. Local maxima and minima, and their connection to derivatives. Mean value theorems: Rolle, Cauchy. L'Hospital rule. Darboux property of the derivative. Higher order derivatives, Taylor polynomials, Taylor series. Specific Taylor series of well-known functions. Convex and concave functions and their connection to second derivatives. Derivative of a convex differentiable function is continuous. Jensen inequality, inequality of various means, Cauchy-Schwarz, Holder inequalities. Plotting functions by analysis of derivatives.

8. Indefinite integrals

   Definition, and elementary integrals. Integration by parts, and by substitution. Partial fraction decomposition, integration of rational functions. Integration of trigonometric, hyperbolic functions.

9. Definite (Riemann) integrals

   Riemann approximation sums, oscillation sums, upper and lower integral. Riemann integrability of a function, sum and products of integrable functions. Newton-Leibniz formula. The integral function. Continuous or monotonic functions are integrable.

10. Applications of the integral, improper integrals

    Arc-length, area. Volume and surface of a body of rotation. Center of gravity. Improper integrals, majorant and minorant criteria.