INTEGRÁLSZÁMÍTÁS

Alapintegrálok

1. $\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$
2. $\int \frac{1}{x} dx = \ln|x| + C$
3. $\int \sin x dx = -\cos x + C$
4. $\int \cos x dx = \sin x + C$
5. $\int \frac{1}{\cos^2 x} dx = tgx + C$
6. $\int \frac{1}{\sin^2 x} dx = -ctgx + C$
7. $\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + C$
8. $\int \frac{1}{1+x^2} dx = arctgx + C$
9. $\int e^x dx = e^x + C$
10. $\int a^x dx = \frac{a^x}{\ln a} + C \qquad (0 < a \neq -1)$

Alapintegrálokra visszavezethető feladatok

1. $\int (x^5 + \frac{1}{x^3}) dx = \int (x^5 + x^{-3}) dx = \frac{x^6}{6} + \frac{x^{-2}}{-2} + C$

2. $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right) dx = \int (x^{\frac{1}{2}} + x^{-\frac{1}{2}}) dx = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C = \frac{2}{3}\sqrt{x^3} + 2\sqrt{x} + C$

3. $\int \left(\sqrt[3]{x^2} + \frac{1}{\sqrt[4]{x}} \right) dx = \int (x^{\frac{2}{3}} + x^{-\frac{1}{4}}) dx = \frac{x^{\frac{5}{3}}}{\frac{5}{3}} + \frac{x^{\frac{3}{4}}}{\frac{3}{4}} + C = \frac{3}{5}\sqrt[3]{x^5} + \frac{4}{3}\sqrt[4]{x^3} dx$

4. $\int \sqrt{x} \cdot \sqrt[3]{x} dx = \int x^{\frac{1}{2}} \cdot x^{\frac{1}{3}} dx = \int x^{\frac{3+2}{6}} dx = \int x^{\frac{5}{6}} dx = \frac{x^{\frac{11}{6}}}{\frac{11}{6}} + C = \frac{6}{11}\sqrt[6]{x^{11}} + C$

5. $\int \frac{\sqrt{x}}{\sqrt[3]{x}} dx = \int \frac{x^{\frac{1}{2}}}{x^{\frac{1}{3}}} dx = \int x^{\frac{3-2}{6}} dx = \int x^{\frac{1}{6}} dx = \frac{x^{\frac{7}{6}}}{\frac{7}{6}} + C = \frac{6}{7}\sqrt[6]{x^7} + C$

6. $\int \sqrt{x \cdot \sqrt[3]{x}} dx = \int (x \cdot x^{\frac{1}{3}})^{\frac{1}{2}} dx = \int (x^{\frac{4}{3}})^{\frac{1}{2}} dx = \int x^{\frac{2}{3}} dx = \frac{x^{\frac{5}{3}}}{\frac{5}{3}} + C = \frac{3}{5}\sqrt[3]{x^5} + C$

7. $\int \frac{x^2 - 7x + 8}{x^2} dx = \int \left(1 - \frac{7}{x} + \frac{8}{x^2} \right) dx = x - 7\ln|x| - \frac{8}{x} + C$

8. $\int \frac{x^3 - 4x^2}{\sqrt{x}} dx = \int \frac{x^3 - 4x^2}{x^{\frac{1}{2}}} dx = \int (x^{\frac{5}{2}} - 4x^{\frac{3}{2}}) dx = \frac{x^{\frac{7}{2}}}{\frac{7}{2}} - 4\frac{x^{\frac{5}{2}}}{\frac{5}{2}} + C = \frac{2}{7}\sqrt{x^7} - 4 \cdot \frac{2}{5} \cdot \sqrt{x^5} + C$

9. $\int (5 \cdot 2^x + 4\sin x - 3\cos x) dx = 5 \cdot \frac{2^x}{\ln 2} - 4\cos x - 3\sin x + C$

10. $\int\left(\frac{3}{\cos^2 x}-\frac{7}{5\sin^2 x}\right)dx = 3tgx+\frac{7}{5}ctgx+C$

11. $\int\frac{-4}{3+3x^2}dx = -\frac{4}{3}\int\frac{1}{1+x^2}dx = -\frac{4}{3}arctgx+C$

12. $\int\frac{x^2}{x^2+1}dx = \int\frac{x^2+1-1}{x^2+1}dx = \int\left(1-\frac{1}{x^2+1}\right)dx = x-arctgx+C$

13. $\int\frac{5}{3\sqrt{4-4x^2}}dx = \frac{5}{3\cdot 2}\int\frac{1}{\sqrt{1-x^2}}dx = \frac{5}{6}\arcsin x+C$

14. $\int tg^2 x dx = \int\frac{\sin^2 x}{\cos^2 x}dx = \int\frac{1-\cos^2 x}{\cos^2 x}dx = \int\left(\frac{1}{\cos^2 x}-1\right)dx = tgx-x+C$

15. $\int\frac{\cos^2 x-5}{1+\cos 2x}dx = \int\frac{\cos^2 x-5}{(\sin^2 x+\cos^2 x)+(\cos^2 x-\sin^2 x)}dx = \int\frac{\cos^2 x-5}{2\cos^2 x}dx =$

$$= \int\left(\frac{1}{2}-\frac{5}{2\cos^2 x}\right)dx = \frac{1}{2}x-\frac{5}{2}tgx+C$$

16. $\int\frac{\cos 2x}{\sin x+\cos x}dx = \int\frac{\cos^2 x-\sin^2 x}{\sin x+\cos x}dx = \int\frac{(\cos x-\sin x)\cdot(\cos x+\sin x)}{\sin x+\cos x}dx =$

$$= \int(\cos x-\sin x)dx = \sin x+\cos x+C$$

Integrálási módszerek

I. $\int f(ax+b)dx = \frac{F(ax+b)}{a}$, ahol $F' = f$.

Feladatok

1. a) $\int(2x+9)^3 dx = \frac{(2x+9)^4}{4\cdot 2}+C$

1. b) $\int(11x-12)^{13} dx = \frac{(11x-12)^{14}}{11\cdot 14}+C$

2. $\int\frac{1}{(7x+1)^4}dx = \int(7x+1)^{-4}dx = \frac{(7x+1)^{-3}}{-3\cdot 7}+C$

3. a) $\int\sqrt{5x-8}dx = \int(5x-8)^{\frac{1}{2}}dx = \frac{(5x-8)^{\frac{3}{2}}}{\frac{3}{2}\cdot 5}+C$

3. b) $\int \sqrt[3]{1-2x}dx = \int (1-2x)^{\frac{1}{3}}dx = \frac{(1-2x)^{\frac{4}{3}}}{\frac{4}{3}\cdot(-2)} + C$

3. c) $\int (4x+2)^{\frac{2}{3}}dx = \frac{(4x+2)^{\frac{5}{3}}}{\frac{5}{3}\cdot 4} + C$

4. $\int \frac{1}{\sqrt{3-4x}}dx = \int (3-4x)^{-\frac{1}{2}}dx = \frac{(3-4x)^{\frac{1}{2}}}{\frac{1}{2}\cdot(-4)} + C$

5. a) $\int \sin 6x dx = \frac{-\cos 6x}{6} + C$

5. b) $\int \cos(-4-5x)dx = \frac{\sin(-4-5x)}{-5} + C$

6. a) $\int \frac{1}{\sin^2\left(2x+\frac{\pi}{4}\right)}dx = -\frac{1}{2}ctg\left(2x+\frac{\pi}{4}\right) + C$

6. b) $\int \frac{5}{\cos^2 3x}dx = \frac{5}{3}tg3x + C$

7. a) $\int \frac{1}{1+3x^2}dx = \int \frac{1}{1+(\sqrt{3}x)^2}dx = \frac{arctg(\sqrt{3}x)}{\sqrt{3}} + C$

7. b) $\int \frac{-2}{20x^2+5}dx = -\frac{2}{5}\int \frac{1}{4x^2+1}dx = -\frac{2}{5}\int \frac{1}{1+(2x)^2}dx = -\frac{2}{5}\frac{arctg 2x}{2} + C$

7. c) $\int \frac{1}{x^2+2x+2}dx = \int \frac{1}{(x+1)^2+1}dx = arctg(x+1) + C$

8. $\int \frac{1}{\sqrt{1-25x^2}}dx = \int \frac{1}{\sqrt{1-(5x)^2}}dx = \frac{\arcsin(5x)}{5} + C$

9. a) $\int e^{6x+5}dx = \frac{e^{6x+5}}{6} + C$

9.b) $\int (e^{-x} + e^{-2x})dx = -e^{-x} - \frac{1}{2}e^{-2x} + C$

10. $\int 5^{3-11x}dx = \frac{5^{3-11x}}{-11}\cdot\frac{1}{\ln 5} + C$

11. a) $\int \frac{1}{8x-3}dx = \frac{\ln|8x-3|}{8} + C$

11. b) $\int \frac{x+1}{x-1}dx = \int \frac{x-1+2}{x-1}dx = \int\left(1+\frac{2}{x-1}\right)dx = x + 2\ln|x-1| + C$

12. a) $\int \sin^2 x dx = \int \frac{1}{2}(1-\cos 2x)dx = \int\left(\frac{1}{2} - \frac{\cos 2x}{2}\right)dx = \frac{1}{2}x - \frac{\sin 2x}{2\cdot 2} + C$

12. b) $\int \cos^2 x dx = \int \frac{1}{2}(1+\cos 2x)dx = \int\left(\frac{1}{2} + \frac{\cos 2x}{2}\right)dx = \frac{1}{2}x + \frac{\sin 2x}{2\cdot 2} + C$

12. c) $\int \sin^2 2x dx = \int \frac{1}{2}(1-\cos 4x)dx = \int \left(\frac{1}{2} - \frac{\cos 4x}{2}\right)dx = \frac{1}{2}x - \frac{\sin 4x}{2\cdot 4} + C$

II. $\int f^n(x) f'(x) dx = \frac{f^{n+1}(x)}{n+1} + C \qquad (n \neq -1)$.

Feladatok

1. a) $\int 8x(1+4x^2)^3 dx = \frac{(1+4x^2)^4}{4} + C$

1. b) $\int x^2(2x^3+9)dx = \frac{1}{6}\int 6x^2(2x^3+9)dx = \frac{1}{6}\frac{(2x^3+9)^2}{2} + C$

1. c) $\int x^2(x^3-2)^5 dx = \frac{1}{3}\int 3x^2(x^3-2)^5 dx = \frac{1}{3}\frac{(x^3-2)^6}{6} + C$

2. a) $\int x^2\sqrt{6x^3+7}dx = \frac{1}{18}\int 18x^2(6x^3+7)^{\frac{1}{2}} dx = \frac{1}{18}\frac{(6x^3+7)^{\frac{3}{2}}}{\frac{3}{2}} + C$

2. b) $\int x\cdot\sqrt[4]{2-3x^2}dx = -\frac{1}{6}\int -6x(2-3x^2)^{\frac{1}{4}}dx = -\frac{1}{6}\frac{(2-3x^2)^{\frac{5}{4}}}{\frac{5}{4}} + C$

3. a) $\int \frac{x}{\sqrt{x^2+1}}dx = \int x(x^2+1)^{-\frac{1}{2}}dx = \frac{1}{2}\int 2x(x^2+1)^{-\frac{1}{2}}dx = \frac{1}{2}\frac{(x^2+1)^{\frac{1}{2}}}{\frac{1}{2}} + C = \sqrt{x^2+1} + C$

3. b) $\int \frac{x}{\sqrt{(x^2+1)^3}}dx = \int x(x^2+1)^{-\frac{3}{2}}dx = \frac{1}{2}\int 2x(x^2+1)^{-\frac{3}{2}}dx = \frac{1}{2}\frac{(x^2+1)^{-\frac{1}{2}}}{-\frac{1}{2}} + C = -\frac{1}{\sqrt{x^2+1}} + C$

3. c) $\int \frac{7x^2}{\sqrt{5-4x^3}}dx = \int 7x^2(5-4x^3)^{-\frac{1}{2}}dx = -\frac{7}{12}\int -12x^2(5-4x^3)^{-\frac{1}{2}}dx = -\frac{7}{12}\frac{(5-4x^3)^{\frac{1}{2}}}{\frac{1}{2}} + C$

3. d) $\int \frac{2x-5}{\sqrt[3]{(x^2-5x+13)^7}}dx = \int (2x-5)(x^2-5x+13)^{-\frac{7}{3}}dx = \frac{(x^2-5x+13)^{-\frac{4}{3}}}{-\frac{4}{3}} + C$

4. a) $\int \sin x\cos x dx = \int \sin x\cdot(\sin x)' dx = \frac{\sin^2 x}{2} + C$

4. b) $\int \cos^3 x\sin x dx = -\int \cos^3 x\cdot(-\sin x)dx = -\frac{\cos^4 x}{4} + C$

4. c) $\int \sin^4 x\sin 2x dx = \int \sin^4 x 2\sin x\cos x dx = 2\int \sin^5 x\cos x dx = 2\frac{\sin^6 x}{6} + C = \frac{1}{3}\sin^6 x + C$

4. d) $\int \frac{\sin x}{\sqrt[3]{\cos^2 x}}dx = \int \sin x(\cos x)^{-\frac{2}{3}}dx = -\int(-\sin x)(\cos x)^{-\frac{2}{3}}dx = -\frac{(\cos x)^{\frac{1}{3}}}{\frac{1}{3}} + C = -3\cdot\sqrt[3]{\cos x} + C$

5. $\int \sin^3 x dx = \int \sin x \sin^2 x dx = \int \sin x(1-\cos^2 x)dx = \int(\sin x - \sin x\cos^2 x)dx = -\cos x + \frac{\cos^3 x}{3} + C$

6. a) $\int \frac{\ln x}{x}dx = \int \frac{1}{x}\ln x dx = \frac{\ln^2 x}{2} + C$

6. b) $\int \frac{\ln^3 x}{x}dx = \int \frac{1}{x}\ln^3 x dx = \frac{\ln^4 x}{4} + C$

6. c) $\int \frac{1}{x\ln^3 x}dx = \int \frac{1}{x}\ln^{-3} x dx = \frac{\ln^{-2} x}{-2} + C$

7. a) $\int \frac{\sin^5 x}{\cos^7 x}dx = \int \frac{\sin^5 x}{\cos^5 x}\cdot\frac{1}{\cos^2 x}dx = \int tg^5 x\frac{1}{\cos^2 x}dx = \frac{tg^6 x}{6} + C$

7. b) $\int \frac{1}{\cos^2 x\sqrt{tgx}}dx = \int \frac{1}{\cos^2 x}(tgx)^{-\frac{1}{2}}dx = \frac{(tgx)^{\frac{1}{2}}}{\frac{1}{2}} + C = 2\sqrt{tgx} + C$

8. $\int \frac{(arctgx)^2}{1+x^2}dx = \int \frac{1}{1+x^2}(arctgx)^2 dx = \frac{(arctgx)^3}{3} + C$

9. $\int \frac{1}{\sqrt{(1-x^2)\arcsin x}}dx = \int \frac{1}{\sqrt{1-x^2}}(\arcsin x)^{-\frac{1}{2}}dx = \frac{(\arcsin x)^{\frac{1}{2}}}{\frac{1}{2}} + C = 2\sqrt{\arcsin x} + C$

10. $\int e^x(1-e^x)^3 dx = -\int -e^x(1-e^x)^3 dx = -\frac{(1-e^x)^4}{4} + C$

III. $\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + C$

Feladatok

1. $\int \frac{2x}{x^2+7}dx = \ln(x^2+7) + C$

2. $\int \frac{5x^2}{x^3+4}dx = \frac{5}{3}\int \frac{3x^2}{x^3+4}dx = \frac{5}{3}\ln|x^3+4| + C$

3. $\int \frac{x^3}{x^4+5}dx = \frac{1}{4}\int \frac{4x^3}{x^4+5}dx = \frac{1}{4}\ln(x^4+5) + C$

4. $\int \frac{x-3}{x^2-6x+10}dx = \frac{1}{2}\int \frac{2x-6}{x^2-6x+10}dx = \frac{1}{2}\ln(x^2-6x+10) + C$

5. a) $\int ctgx dx = \int \frac{\cos x}{\sin x} dx = \ln|\sin x| + C$

5. b) $\int tgx dx = \int \frac{\sin x}{\cos x} dx = -\int \frac{-\sin x}{\cos x} dx = -\ln|\cos x| + C$

6. a) $\int \frac{\cos x}{3\sin x + 1} dx = \frac{1}{3} \int \frac{3\cos x}{3\sin x + 1} dx = \frac{1}{3} \ln|3\sin x + 1| + C$

6. b) $\int \frac{4\sin x}{5\cos x + 4} dx = -\frac{4}{5} \int \frac{-5\sin x}{5\cos x + 4} dx = -\frac{4}{5} \ln|5\cos x + 4| + C$

7. $\int \frac{\sin 2x}{\sin^2 x + \pi} dx = \int \frac{2\sin x \cos x}{\sin^2 x + \pi} dx = \ln(\sin^2 x + \pi) + C$

8. a) $\int \frac{1}{\cos^2 x tgx} dx = \int \frac{\frac{1}{\cos^2 x}}{tgx} dx = \ln|tgx| + C$

8. b) $\int \frac{1}{\sin^2 x ctgx} dx = -\int \frac{-\frac{1}{\sin^2 x}}{ctgx} dx = -\ln|ctgx| + C$

9. $\int \frac{1}{(1+x^2) arctgx} dx = \int \frac{\frac{1}{1+x^2}}{arctgx} dx = \ln|arctgx| + C$

10. $\int \frac{1}{\sqrt{1-x^2} \arcsin x} dx = \int \frac{\frac{1}{\sqrt{1-x^2}}}{\arcsin x} dx = \ln|\arcsin x| + C$

11. $\int \frac{1}{x \ln x} dx = \int \frac{\frac{1}{x}}{\ln x} dx = \ln|\ln x| + C$

12. $\int \frac{e^{2x}}{e^{2x} + 3} dx = \frac{1}{2} \int \frac{2e^{2x}}{e^{2x} + 3} dx = \frac{1}{2} \ln(e^{2x} + 3) + C$

IV. Parciális integrálás

$$\int u'(x) \cdot v(x) dx = u(x)v(x) - \int u(x)v'(x) dx$$

Feladatok

1. $\int \underset{v}{x} \cdot \underset{u'}{e^x} dx = x \cdot e^x - \int e^x dx = x \cdot e^x - e^x + C$

2. $\int \underset{v}{x} \cdot \underset{u'}{e^{2x}} dx = x \cdot \frac{e^{2x}}{2} - \int \frac{e^{2x}}{2} dx = x \cdot \frac{e^{2x}}{2} - \frac{e^{2x}}{4} + C$

3. $\int \underset{v}{x^2}\cdot \underset{u'}{e^{2x+4}}\,dx = x^2\cdot\frac{e^{2x+4}}{2}-\int \underset{v}{2x}\cdot\underset{u'}{\frac{e^{2x+4}}{2}}dx = x^2\cdot\frac{e^{2x+4}}{2}-\left[2x\cdot\frac{e^{2x+4}}{4}-\int 2\cdot\frac{e^{2x+4}}{4}dx\right]=$

$= x^2\cdot\frac{e^{2x+4}}{2}-2x\cdot\frac{e^{2x+4}}{4}+2\cdot\frac{e^{2x+4}}{8}+C$

4. $\int \underset{v}{x}\cdot \underset{u'}{\sin 5x}\,dx = x\cdot\left(-\frac{\cos 5x}{5}\right)-\int -\frac{\cos 5x}{5}dx = -x\cdot\frac{\cos 5x}{5}+\frac{\sin 5x}{25}+C$

5. $\int \underset{v}{x^2}\cdot \underset{u'}{\cos x}\,dx = x^2\sin x-\int \underset{v}{2x}\cdot \underset{u'}{\sin x}dx = x^2\sin x-\left[2x(-\cos x)-\int 2\cdot(-\cos x)dx\right]=$

$= x^2\sin x+2x\cos x-2\sin x+C$

6. $x\in \mathrm{R}^+$, $\int \ln x dx = \int \underset{u'}{1}\cdot \underset{v}{\ln x}\,dx = x\ln x-\int x\cdot\frac{1}{x}dx = x\ln x - x + C$

7. $x\in \left]-1,1\right[$, $\int \arcsin x dx = \int \underset{u'}{1}\cdot \underset{v}{\arcsin x}\,dx = x\arcsin x-\int x\frac{1}{\sqrt{1-x^2}}dx =$

$= x\arcsin x+\frac{1}{2}\int -2x\cdot(1-x^2)^{-\frac{1}{2}}dx = x\arcsin x+\frac{1}{2}\frac{(1-x^2)^{\frac{1}{2}}}{\frac{1}{2}}+C$

$= x\arcsin x+\sqrt{1-x^2}+C$

8. $\int \mathrm{arctg}\, 3x dx = \int \underset{u'}{1}\cdot \mathrm{arctg}3x\,dx = x\mathrm{arctg}3x-\int x\cdot\frac{1}{\underset{f'/f}{1+9x^2}}\cdot 3dx =$

$= x\mathrm{arctg}3x-\frac{1}{6}\int 18x\cdot\frac{1}{1+9x^2}dx = x\mathrm{arctg}3x-\frac{1}{6}\ln\left(1+9x^2\right)+C$

9. $x\in\left]-\frac{3}{5},\infty\right[$, $\int \underset{v}{x}\cdot \underset{u'}{\sqrt{5x+3}}\,dx =$

$= x\cdot\frac{(5x+3)^{\frac{3}{2}}}{\frac{3}{2}\cdot 5}-\int 1\cdot\frac{(5x+3)^{\frac{3}{2}}}{\frac{3}{2}\cdot 5}dx = \frac{2}{15}x\cdot(5x+3)^{\frac{3}{2}}-\frac{2}{15}\cdot\frac{(5x+3)^{\frac{5}{2}}}{\frac{5}{2}\cdot 5}+C$

10. $x\in \mathrm{R}^+$, $\int\frac{\ln x}{\sqrt{x}}dx = \int \underset{u'}{\frac{1}{\sqrt{x}}}\cdot \underset{v}{\ln x}\,dx = \frac{x^{\frac{1}{2}}}{\frac{1}{2}}\ln x-\int\frac{x^{\frac{1}{2}}}{\frac{1}{2}}\cdot\frac{1}{x}dx =$

$= 2x^{\frac{1}{2}}\ln x-\int 2x^{-\frac{1}{2}}dx = 2x^{\frac{1}{2}}\ln x-2\frac{x^{\frac{1}{2}}}{\frac{1}{2}}+C = 2\sqrt{x}\ln x-4\sqrt{x}+C$

11. $\int \ln\left(x^2+1\right)dx = \int \underset{u'}{1}\cdot \underset{v}{\ln\left(x^2+1\right)}dx = x\ln\left(x^2+1\right)-\int x\cdot\frac{1}{x^2+1}\cdot 2x\,dx =$

$= x\ln\left(x^2+1\right)-\int\frac{2x^2+2-2}{x^2+1}dx = x\ln\left(x^2+1\right)-\int\left(2-\frac{2}{x^2+1}\right)dx =$

$= x\ln\left(x^2+1\right)-2x+2arctgx+C$

V. $\int f\big(g(x)\big)\cdot g'(x)dx = F\big(g(x)\big)+C$, ahol $F' = f$.

Feladatok

1. a) $\int 2x\cos(x^2+1)dx = \sin(x^2+1)+C$

1. b) $\int x\sin x^2 dx = -\frac{1}{2}\cos x^2 + C$

1. c) $\int (3x^2+2)\sin(x^3+2x-4)dx = -\cos(x^3+2x-4)+C$

2. a) $\int e^{\sin x}\cos x dx = e^{\sin x}+C$

2. b) $\int \frac{1}{\cos^2 x}e^{tgx}dx = e^{tgx}+C$

2. c) $\int xe^{-x^2}dx = -\frac{1}{2}\int -2xe^{-x^2}dx = -\frac{1}{2}e^{-x^2}+C$

3. a) $\int \frac{2x}{\sqrt{1-x^4}}dx = \int 2x\frac{1}{\sqrt{1-(x^2)^2}}dx = \arcsin x^2 + C$

3. b) $\int \frac{e^x}{\sqrt{1-e^{2x}}}dx = \int e^x\frac{1}{\sqrt{1-(e^x)^2}}dx = \arcsin(e^x)+C$

4. a) $\int \frac{2x}{1+x^4}dx = \int 2x\frac{1}{1+(x^2)^2}dx = arctgx^2 + C$

4. b) $\int \frac{e^x}{1+e^{2x}}dx = arctg(e^x)+C$

4. c) $\int \frac{1}{x(1+\ln^2 x)}dx = \int \frac{1}{x}\cdot\frac{1}{1+(\ln x)^2}dx = arctg(\ln x)+C$

4. d) $\int \frac{\cos x}{1+\sin^2 x}dx = arctg(\sin x)+C$