
Calculus 1 - Homework 3

1. (4 points) For what values of x does the power series $\sum_{n=1}^{\infty} (-1)^n \frac{(2x-3)^{2n}}{4^{n+1}}$ converge?

Evaluate the sum.

2. (4 points) Let $A = (-3, -1] \cup \left\{ \frac{2}{n} : n \in \mathbb{N} \right\} \cup \{3\} \cup [5, 6] \subset \mathbb{R}$.

Find the set of interior points, boundary points, limit points and isolated points of A .

3. (4 points) $\lim_{x \rightarrow 4} \frac{\sqrt{1+2x} - 3}{\sqrt{x} - 2} = ?$

4. (5 points) $\lim_{x \rightarrow 0} \frac{\sin^2(2x)}{\cos(3x) - 1} = ?$

5. (4 points) Choose the values of the parameters so that the following functions be continuous on \mathbb{R} :

$$f(x) = \begin{cases} (x-1)^3 & \text{if } x \leq 0 \\ ax + b & \text{if } 0 < x < 1 \\ \sqrt{x} & \text{if } x \geq 1 \end{cases}$$

6. (5 points) Determine the points of discontinuity of the following function.

What type of discontinuities are these?

$$f(x) = \frac{|x-2| \sin(x-1)}{x^2 - 3x + 2}$$

7. (4 points) Let $f(x) = 2 \sin x - x^2 + 1$. Prove that f has a zero in the open interval $(0, 2)$.

Deadline: November 18th