
Calculus 1 - Homework 1

1. Prove by induction that $2^n > n^2$ for $n \geq n_0$. Find the smallest such positive integer n_0 . **(4 points)**

2. Find the minimum value of $f(x, y) = x^2 + y^2 + \frac{2}{xy}$ if $x > 0, y > 0$. **(4 points)**

3. Find the infimum and supremum of the set $H = \{n^{(-1)^n} : n \in \mathbb{N}\}$. **(3 points)**

4. Find the algebraic form of $\frac{z^2 - |z|^2}{z - \bar{z}}$ if $z = \sqrt{3} + i$. **(3 points)**

5. Give all the solutions of the following equation in algebraic form:

$$iz^3 = \frac{1}{2}(1 - i)^8. \quad \mathbf{(4 \text{ points})}$$

6. Let $a_n = \frac{8n^2 + 3n - 5}{4n^2 - n + 4}$. Find the limit of a_n and provide a threshold index N for $\varepsilon = 0.01$. **(4 points)**

7. Find the limit of the sequence $a_n = \frac{1}{n^2 - \sqrt{n^4 - 5n^2}}$. **(3 points)**

Deadline: September 28th