Practice exercises 8.

1. Using the definition of the limit, prove the following equalities:

a) $\lim_{x\to 1}(3x+4)=7$ b) $\lim_{x\to -2}\frac{8-2x^2}{x+2}=8$ c) $\lim_{x\to -3}\sqrt{1-5x}=4$

d) $\lim_{x\to 1}\frac{1}{(1-x)^2}=\infty$ e) $\lim_{x\to \infty}\frac{1-2x}{x+3}=-2$ f) $\lim_{x\to -\infty}\frac{1-2x}{x+3}=-2$

2. Calculate the following limits:

a) $\lim_{x\to 3}\frac{x^2-2x-3}{x^2-5x+6}$ b) $\lim_{x\to -2}\frac{x^2+3x-10}{(x^2-4)^2}$ c) $\lim_{x\to 2}\frac{x^2+3x-10}{(x^2-4)^2}$

d) $\lim_{x\to -\infty}\left(\frac{x^3+3x^2}{x^2+1}-x\right)$ e) $\lim_{x\to \infty}\frac{\sqrt{x}+\sqrt[3]{x}+\sqrt[4]{x}}{\sqrt{2x+1}}$ f) $\lim_{x\to \infty}\left(\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\right)$

g) $\lim_{x\to -\infty}x\left(\sqrt{x^2+1}-\sqrt{x^2+3}\right)$ h) $\lim_{x\to 6}\frac{\sqrt{x-2}-2}{x-6}$ i) $\lim_{x\to 5}\frac{2-\sqrt{x-1}}{x^2-25}$

j) $\lim_{x\to 1}\frac{1-x^2}{\sqrt{x}-\sqrt{2-x}}$ k) $\lim_{x\to 1}\left(\frac{2}{1-x^2}-\frac{3}{1-x^3}\right)$ l) $\lim_{x\to 1}\left(\frac{2}{\sqrt[3]{x}-1}-\frac{3}{\sqrt{x}-1}\right)$

m) $\lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1+x^2}}{\sqrt{1+x}-1}$ n) $\lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt[3]{1+x}-\sqrt[3]{1-x}}$ o) $\lim_{x\to -8}\frac{\sqrt{1-x}-3}{2+\sqrt[3]{x}}$

3. Prove that $\lim_{x\to 0}\frac{\sin x}{x}=1$.

4. Using the above result, calculate the following limits:

a) $\lim_{x\to 0}\frac{\sin(2x)}{3x}$ b) $\lim_{x\to 0}\frac{\tan(3x)}{\sin(7x)}$ c) $\lim_{x\to 0}\left(\frac{1}{\sin x}-\frac{1}{\tan x}\right)$

d) $\lim_{x\to 0}\frac{1-\sqrt{1+2x}}{\sin x}$ e) $\lim_{x\to 0}\frac{1-\cos x}{x^2}$ f) $\lim_{x\to 0}\frac{-1+\cos 3x}{7x^2}$

g) $\lim_{x\to 0}\frac{1-\cos 5x}{x\sin 3x}$ h) $\lim_{x\to 0}\frac{1-\cos 4x}{\tan 3x\sin 7x}$ i) $\lim_{x\to 0}\frac{\sqrt{1-\cos(2x)}}{x}$

j) $\lim_{x\to 1}(x-1)\tan\left(\frac{\pi x}{2}\right)$ k) $\lim_{x\to 0}\frac{1-\sqrt{\cos x}}{1-\cos\left(\sqrt{x}\right)}$ l) $\lim_{x\to 0}\frac{\sin^2(5x)}{\cos(4x)-\cos(6x)}$

5.* Calculate the following limits:

a) $\lim_{x\to 0}\left(\frac{1+\tan x}{1+\sin x}\right)^{\frac{1}{\sin x}}$ b) $\lim_{x\to 0}\left(\sqrt{1+x}-x\right)^{\frac{1}{x}}$ c) $\lim_{x\to \frac{\pi}{2}}(\sin x)^{\tan x}$