
Practice exercises 7.

Topology of the real numbers

1. Answer the following questions.

- (i) Find the set of interior points ($\text{int } H$), boundary points (∂H), limit points (H') and isolated points of H .
- (ii) Is the set H open, closed or neither? Is the set H bounded?
- (iii) Find the closure of H (it is denoted by \overline{H}). Is the set H compact?

a) $H = \mathbb{Z}$

b) $H = \mathbb{Q}$

c) $H = \mathbb{R} \setminus \mathbb{Q}$

d) $H = (-2, -1) \cup [3, 5] \cup \{7\} \cup [8, \infty)$

e) $H = \left\{ \frac{1}{n} : n \in \mathbb{N}^+ \right\}$

f) $H = \{0\} \cup \left\{ \frac{1}{n} : n \in \mathbb{N}^+ \right\}$

g) $H = \mathbb{Q} \cap [0, 1]$

h) $H = \bigcup_{n=1}^{\infty} \left[\frac{1}{2n+1}, \frac{1}{2n} \right]$

2. Prove that for any real values a_1, \dots, a_n the finite set $\{a_1, \dots, a_n\} \subset \mathbb{R}$ is closed.

3. Let $A \subset \mathbb{R}$. Prove that

- a) the set of interior points of A is open;
- b) the set of boundary points of A is closed;
- c) the set of limit points of A is closed.

4. Show an example of a set $A \subset \mathbb{R}$ for which $\text{int } \overline{A} = \mathbb{R}$ and $\overline{\text{int } A} = \emptyset$.

- 5. a) Give an example of infinitely many open sets such that their intersection is closed.
- b) Give an example of infinitely many closed sets such that their union is open.

6. Show an example of an open cover of the interval $(0, 1)$ from which a finite subcover cannot be selected.

7. Prove that if G is open and F is closed then $G \setminus F$ is open and $F \setminus G$ is closed.