

Practice exercises 6.

1. Decide whether the following series are convergent or divergent.

$$\text{a) } \sum_{n=1}^{\infty} \frac{10^n}{n!}$$

$$\text{b) } \sum_{n=1}^{\infty} \frac{n^4}{2^n}$$

$$\text{c) } \sum_{n=1}^{\infty} \frac{3^n}{n^3 2^n}$$

$$\text{d) } \sum_{n=1}^{\infty} \frac{2^n n^2}{n!}$$

$$\text{e) } \sum_{n=1}^{\infty} \frac{n}{(\ln n)^n}$$

$$\text{f) } \sum_{n=1}^{\infty} \frac{(n!)^n}{(n^n)^2}$$

$$\text{g) } \sum_{n=1}^{\infty} \frac{(n!)^n}{n(n^n)}$$

$$\text{h) } \sum_{n=1}^{\infty} \frac{n \log n}{2^n}$$

$$\text{i) } \sum_{n=1}^{\infty} \frac{n^n}{(n!)^2}$$

$$\text{j) } \sum_{n=1}^{\infty} \frac{2^n \cdot n!}{n^n}$$

$$\text{k) } \sum_{n=1}^{\infty} \frac{1}{n!} \left(\frac{n}{e-1} \right)^n$$

$$\text{l) } \sum_{n=1}^{\infty} \left(\frac{n}{n^2+1} \right)^{n^2}$$

$$\text{m) } \sum_{n=1}^{\infty} \left(\frac{n+1}{n+2} \right)^{2n^2+1}$$

$$\text{n) } \sum_{n=1}^{\infty} \left(\frac{4n}{4n+1} \right)^{3n^2}$$

$$\text{o) } \sum_{n=1}^{\infty} \frac{(n+2)^n}{(n+1)!}$$

$$\text{p) } \sum_{n=1}^{\infty} \frac{(3n)!}{(n!)^3}$$

$$\text{q) } \sum_{n=1}^{\infty} \frac{4^n (n!)^2}{(2n)!}$$

$$\text{r) } \sum_{n=1}^{\infty} \frac{(n!)^2 - 2^n}{(2n)!}$$

$$\text{s) } \sum_{n=1}^{\infty} \frac{n^4 + n \log n - 2^n}{n! + n^{10} + 3^n}$$

$$\text{t) } \sum_{n=1}^{\infty} \frac{(n!)^3}{(3n)!} \left(1 + \frac{1}{n} \right)^{n^2+1}$$

2*. Convergent or divergent?

$$\frac{1000}{1} + \frac{1000 \cdot 1001}{1 \cdot 3} + \frac{1000 \cdot 1001 \cdot 1002}{1 \cdot 3 \cdot 5} + \dots$$

3. Show that the following series are convergent. Estimate the error if the sum of the series is approximated by the 100th partial sum.

$$\text{a) } \sum_{n=1}^{\infty} (-1)^n \frac{n+1}{n^2+2}$$

$$\text{b) } \sum_{n=1}^{\infty} (-1)^n \frac{5^n}{2^n + 10^n}$$

4. Are the following series convergent or divergent?

$$\text{a) } \sum_{n=1}^{\infty} (-1)^n \frac{\sin\left(\frac{n\pi}{2}\right)}{n}$$

$$\text{b) } \sum_{n=1}^{\infty} (-1)^n \frac{2n-1}{3n}$$

$$\text{c) } \sum_{n=1}^{\infty} \left(\frac{4-n}{4+n} \right)^n$$

$$\text{d) } \sum_{n=2}^{\infty} (-1)^{n+1} \frac{2n}{n^2-1}$$

$$\text{e) } \sum_{n=2}^{\infty} (-1)^n \frac{1}{n-\sqrt{n}}$$

$$\text{f) } \sum_{n=1}^{\infty} (-1)^n (\sqrt{n+1} - \sqrt{n})$$

$$\text{g) } \sum_{n=1}^{\infty} (-1)^n \log\left(1 + \frac{1}{n}\right)$$

$$\text{h) } \sum_{n=1}^{\infty} (-1)^n (\sqrt[n]{n} - 1)$$

5. Are the following series absolutely or conditionally convergent?

$$\text{a) } \sum_{n=1}^{\infty} \frac{(-3)^n}{4^n + n}$$

$$\text{b) } \sum_{n=1}^{\infty} \frac{(-3)^n}{2 + n 3^n}$$

$$\text{c) } \sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n \sqrt{n}}$$

$$\text{d) } \sum_{n=1}^{\infty} \frac{\cos(n\pi)}{\log n}$$

$$\text{e) } \sum_{n=1}^{\infty} \frac{(-1)^n}{2n^2 - 3n + 8}$$

6. Using the product theorem for series evaluate the following sums in closed form:

$$a) \sum_{n=1}^{\infty} n x^n \quad b) \sum_{n=1}^{\infty} n^2 x^n \quad c) \sum_{n=1}^{\infty} (n+1)^3 x^n$$

7. Let $f(x) = \sum_{n=1}^{\infty} \frac{x^n}{n!}$. Using the product theorem for series prove that $f(x+y) = f(x) \cdot f(y)$.

8. For what values of x do the following power series converge? In cases $a)$, $b)$, $c)$ evaluate the sum.

$$\begin{array}{llll}
 a) \sum_{n=1}^{\infty} x^n & b) \sum_{n=1}^{\infty} (x-2)^n & c) \sum_{n=1}^{\infty} 3^{n+1} x^n & d) \sum_{n=1}^{\infty} n x^n \\
 e) \sum_{n=1}^{\infty} n^n x^n & f) \sum_{n=1}^{\infty} \frac{1}{n^2} x^n & g) \sum_{n=1}^{\infty} \frac{x^n}{n!} & h) \sum_{n=1}^{\infty} \frac{2^n}{n!} x^n \\
 i) \sum_{n=1}^{\infty} \frac{(-1)^n}{n \sqrt{n}} (x+3)^n & j) \sum_{n=1}^{\infty} \cos\left(\frac{n\pi}{4}\right) \frac{n+1}{n^2+1} x^n & k) \sum_{n=1}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} \\
 l) \sum_{n=1}^{\infty} \frac{n}{(n+1)3^n} x^{2n} & m) \sum_{n=1}^{\infty} \frac{n+1}{9^n} (x-2)^{2n} & n) \sum_{n=1}^{\infty} \frac{n!}{n^n} x^n
 \end{array}$$