Practise exercises 1.

(1) Prove by induction that $1^2 + 3^2 + ... + (2n-1)^2 = n(2n-1)(2n+1)/3$.

(2) Prove by induction that $\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}$

(3) Prove by induction that for every $n > 1$ we have $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \cdots + \frac{1}{\sqrt{n}} > \sqrt{n}$.

(4) Prove by induction that for all $n > 1$ we have $\frac{(2n)!}{(n!)^2} > \frac{4^n}{n+1}$

(5) Let $a_0 = 1$ and $a_{n+1} = \sqrt{3a_n + 10}$. Prove that the sequence a_n is monotonically increasing.

(6) Let A, B, C be some sets. Using set operations (intersection, union, complement, etc.) define the following sets:

    a; The set of elements of B which are not included in either A or C.

    b; The set of elements which belong to exactly two of the sets A, B, C.

    c; The set of elements which are not included in all of the three sets.

    d; Elements which belong to at most one of the sets.

(7) Write the following statements with logical formulas:

    a; There exists an odd natural number larger than 10.

    b; Every odd number, which is larger than one, is a prime number.

    Write down also the negations of the above statements, both with words and with logical formulas.

(8) Put the following statements into words:

    a; $\forall x \in \mathbb{R}((x > 0) \Rightarrow (\exists k \in \mathbb{N}(2^{-k} < x)))$

    b; $\exists k \in \mathbb{N}(\forall x \in \mathbb{R}((x > 0) \Rightarrow (2^{-k} < x)))$.

    Decide whether the statements are true or false. Write down also the negations of the above statements, both with words and with logical formulas.

(9) Let $P(x)$ mean that x is an even number, and let $H(x)$ mean that x is divisible by 6. Put the following statements into words:

    a; $P(4) \wedge H(12)$

    b; $\forall x(P(x) \Rightarrow H(x))$

    c; $\exists x(P(x) \Rightarrow H(x))$

    d; $\exists x(H(x) \Rightarrow \neg P(x))$

    Decide whether the statements are true or false. Write down also the negations of the above statements, both with words and with logical formulas.