Calculus 1, Final exam, Part 2, 2018.12.18. 10.30-12.00

I indicated the number of points available at each problem.

1. Find the limit of the following sequence: $a_n = (\frac{2n^2+7n-1}{2n^2-n+3})^{3n+1}$ (7 points)

2. Find the limit of the following sequence: $b_n = n^{3/2}(\frac{1}{\sqrt{n+3}} - \frac{1}{\sqrt{n+7}})$ (7 points)

3. Decide whether the following series converges and if so, find its sum: $\sum_{n=2}^{\infty} \frac{3^{n+2}}{5^{n-1}}$ (8 points)

4. Decide whether the following series converges and if so, find its sum: $\sum_{n=1}^{\infty} \frac{n!}{3^n n^6}$ (8 points)

5. Give the Taylor polynomial of order 3 of the function $f(x) = \ln x$ around the point $x_0 = 1$. Approximate the value of $\ln 1.2$ accordingly, and give an error estimate of your approximation. (14 points)

6. Find the following limit: $\lim_{x\to 0} \frac{x^2}{e^{x^2} - \cos 6x}$ (10 points)

7. A rectangle R is inscribed in a semi-circle S of radius 1. What is the maximal possible area of R? (10 points)

8. Calculate the following integral $\int_1^{\infty} \frac{1}{(x+2)(x+4)} dx$ (12 points)

9. Calculate the following integral $\int_0^{\infty} e^{-3x}(x^2+1)dx$ (12 points)

10. Consider the function $f(x) = \frac{\sqrt{\sin x}}{\cos x}$ on the interval $x \in [0, \pi/4]$. Rotate it around the x-axis, and find the volume of the arising body. (12 points)