Name:

No books allowed. Do not share notes. In order to receive the total score, you must show your work and explain your detailed reasoning, except on the "short answer" questions. You have 90 minutes to complete the test. Adding notes from history or informatics results extra points up to +6.

1. (6 points) Prove – by the method of truth tables – that the following inference rule is *valid.*

$$\frac{A \vee B, \quad A \vee (\neg C), \quad (\neg C) \Rightarrow (\neg B)}{C}$$

A	B	C					

2. (2+2+2 points) Determine the quantificational or propositional logical structure of the following sentences and translate them to the symbolic language of logic. Mine the parantheses.

a. „Meg kell értetni a Sárikával
hogy jó lesz vigyázni a bikával
ha jön a bika
ha jön a bika
ha jön a bikák legnagyobbika."

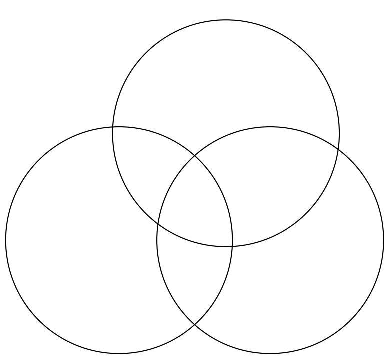
b. „De a kevés is jobb, mint semmi se,
az élet szép, csak bánni kell vele,
s ha félrebillen kedvem kereke,
helyrezökkenti a varázsige:
Fű, fa füst."

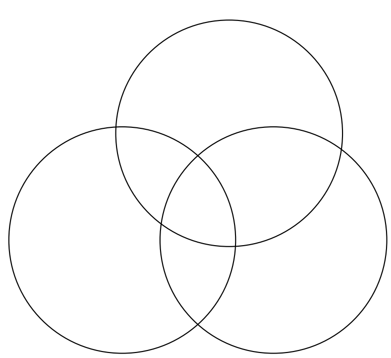
c. „Hogyha bármit ölnek, engem ölnek,
minden kínja, keserve a földnek
rám csap éhesen."

3. (6 points) Prove – by the technique of Venn diagrams – that the following argument is valid.

Premisszák:
1. Akinek leégett a háza, az hajléktalan.
2. Akinek leégett a háza, annak fizet a biztosító.
3. A biztosító egyetlen hajléktalannak sem fizet.

Konklúzió: Senkinek sem égett le a háza.





4. (6 points) Show that $\log_2 6$ is not a rational number.

5. (6 points) Prove by *mathematical induction* that for every positive integer n the expression $3^n - 1$ is divisible by 2 (that is $2 \,|\, (3^n - 1)$ for every $n \in \mathbb{Z}^+$).

6. (4+2 points) Decide whether the following claims are true or false. (If it is true, prove it, if it is false, then prove it or give a counterexample.)

a. There are three white and three red balls in a box. We decrease the number of the balls in the box as follows. We choose three arbitrary balls and perform the following process:
1 if they are composed of rrr, then we throw one red away and put two reds back.
2 if they are composed of rrw, then we put the white and one red back, and we throw the other red away.
3 if they are composed of rww, then we throw the whites away, put the red back and we put a new red into the box.
4 if they are composed of www, then we throw two whites away, put the third white back and we put a new red into the box.
Claim: the 2th step of the process can result: rrrw.

b. All the natural numbers from 1 to 13 are written on a blackboard. At each step of the process two of the numbers are erased and the absolute difference of the couple is written instead.
Claim: It is possible that, after the erasure of the last couple, the process results 1.

7. a) (1+1+1 points) Let R be the following relation in $\mathbb{Z} \times \mathbb{Z}$

$$a\,R\,b \iff |a| \le |b| \qquad ((a,b) \in \mathbb{Z} \times \mathbb{Z})$$

($|a|$ is the absolute value of the real number a.) Write the right answer into the square ($\boxed{\mathsf{T}}$rue or $\boxed{\mathsf{F}}$alse).

R is symmetric	
R is antisymmetric	
R is transitive	

b) (1+1+1 points) Let R be the following relation in $\mathbb{R} \times \mathbb{R}$

$$x\,R\,y \iff y^8 = x \qquad ((x,y) \in \mathbb{R} \times \mathbb{R})$$

Write the right answer into the square ($\boxed{\mathsf{T}}$rue or $\boxed{\mathsf{F}}$alse).

the domain of R is $[0, +\infty)$	
R is a function of x	
R is injective	