Name:

No books allowed. Do not share notes. In order to receive the total score, you must show your work and explain your detailed reasoning, except on the "short answer" questions. You have 90 minutes to complete the test. Adding notes from history or informatics results extra points up to +6.

1. (6 points) Prove – by the method of truth tables – that the following inference rule is valid ("composition rule for alternation").

$$\frac{(A \Rightarrow B) \vee (A \Rightarrow C)}{A \Rightarrow (B \vee C)}$$

A	B	C					

2. (2+2+2 points) Determine the quantificational or propositional logical structure of the following sentences and translate them to the symbolic language of logic. Mine the parantheses.

a. „Minden srác felnéz egy szakszofonosra."

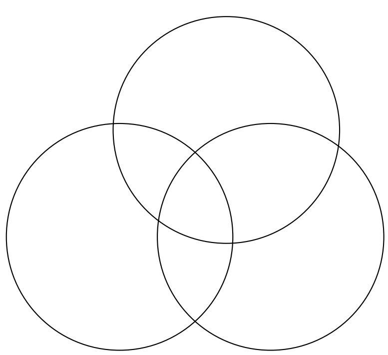
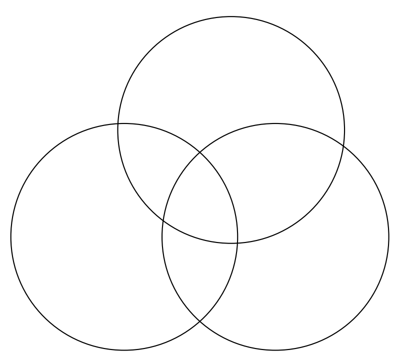
b. „Van olyan város, ahol az utcán mindenki járja a táncot."

c. „Alfonz is megírja a zh-t, ha nem készül el a beadandóval."

3. (6 points) Prove – by the technique of Venn diagrams – that the following argument is valid.

1. A dzsedik közül valaki törölte a bolygó koordinátáit az adatbázisból.
2. Aki nem törölte a koordinátákat, az nem járt a dzsedi templomban.
3. Aki nem járt a dzsedi templomban, az nem dzsedi.

Van valaki, aki járt a dzsedi templomban.

4. (6 points) Show that $\log_{11} 2$ is not a rational number.

5. (6 points) Prove by *mathematical induction* that for every positive integer n the expression $9^n - 2^n$ is divisible by 7 (that is $7 \mid (9^n - 2^n)$ for every $n \in \mathbb{Z}^+$).

6. (2+4 points) *Decide* whether the following claims are true or false. (If it is true, prove it, if it is false, then prove it or give a counterexample.)

a. $(A \cup B) \cap A = A$, for every set A and B .

b. All the natural numbers from 1 to 11 are written on a blackboard. At each step of the process two of the numbers are erased and the difference (always a positive number) of the couple is written instead. It is possible that, after the erasure of the last couple, the process results 2.

7. a) (1+1+1 points) Let ϱ be the following relation in $\mathcal{P}(\{1;2;3;4;5\}) \times \mathcal{P}(\{1;2;3;4;5\})$

$$H \,\varrho\, K \;\Leftrightarrow\; H \subset K \qquad ((H,K) \in \mathcal{P}(\{1;2;3;4;5\}) \times \mathcal{P}(\{1;2;3;4;5\}))$$

($\mathcal{P}(S)$ denotes the set of all subsets of the set S. H is a subset of S, if every member of H is also a member of S.) Write the right answer into the square (T rue or F alse).

ϱ is symmetric	
ϱ is antisymmetric	
ϱ is transitive	

b) (1+1+1 points) Let ϱ be the following relation in $\mathbb{R} \times \mathbb{R}$

$$x \,\varrho\, y \;\Leftrightarrow\; \frac{1}{y^2} = x \qquad ((x,y) \in \mathbb{R} \times \mathbb{R})$$

Write the right answer into the square (T rue or F alse).

the domain of ϱ is $(0,+\infty)$	
ϱ is a function of x	
ϱ is injective	