

# Quantum Information Theory and Mathematical Physics 2019

Budapest, 14-17. June 2022

## Program

	Tuesday 14.06	Wednesday 15.06	Thursday 16.06	Friday 17.06
9.30 – 10.45	Omar Fawzi	Angela Capel	Marco Tomamichel	Richard Kueng
10.45 – 11.15	Coffee break			
11.15 – 12.30	Péter Vrana	Daniel Stilck Franca	Ludovico Lami	Mario Berta
12.30 – 14.30	Lunch			
14.30 – 15.45	Nilanjana Datta	Ivan Todorov	Bartosz Regula	
15.45 – 16.15	Coffee break			
16.15 –	Free discussion, sightseeing, dinner			

### Conference dinner:

Time: Wednesday, 15 June, 19:00-21:30

Location: [Ape Regina](#), 1065 Budapest, Podmaniczky u. 18.

[google map](#)

### Monday Beer:

Time: Monday, 13 June, 19:00-

Location: [Bálna Terasz](#), 1093 Budapest, Fővám tér 11-12.

[google map](#)

# Talks

**Speaker:** Mario Berta

**Title:** Chain rules for quantum channels

**Abstract:** Divergence chain rules for channels relate the divergence of a pair of channel inputs to the divergence of the corresponding channel outputs. An important special case of such a rule is the data-processing inequality, which tells us that if the same channel is applied to both inputs then the divergence cannot increase. Based on direct matrix analysis methods, we derive several Rényi divergence chain rules for channels in the quantum setting. Our results simplify and in some cases generalise previous derivations in the literature.

**Speaker:** Angela Capel

**Title:** Entropy decay for Davies semigroups of a one dimensional quantum lattice

**Abstract:** Given a finite-range, translation-invariant commuting system Hamiltonians on a spin chain, we will show in this talk that the Davies semigroup describing the reduced dynamics resulting from the joint Hamiltonian evolution of a spin chain weakly coupled to a large heat-bath thermalizes rapidly at any temperature. More precisely, we will prove that the relative entropy between any evolved state and the equilibrium Gibbs state contracts exponentially fast with an exponent that scales logarithmically with the length of the chain. This has wide-ranging applications to the study of many-body in and out-of-equilibrium quantum systems.

The key ingredient in the strategy followed in our proof are the so-called results of quasi-factorization of the relative entropy, which we will carefully review in this talk. Moreover, our proof also relies upon a recently derived strong decay of correlations for Gibbs states of one dimensional, translation-invariant local Hamiltonians, and tools from the theory of operator spaces.

**Speaker:** Nilanjana Datta

**Title:** Majorization and entropic continuity bounds

**Abstract:** We employ majorization theory to obtain a powerful tool for deriving simple and universal proofs of continuity bounds for various entropies which are relevant in information theory. In obtaining this, we first derive a more general result which may be of independent interest: a necessary and sufficient condition under which a state maximizes a concave, continuous, Gateaux-differentiable function in an epsilon-ball in trace distance. Examples of such a function include the von Neumann entropy, Renyi entropies, and the conditional entropy. In particular, by introducing a notion of majorization flow, we prove that the  $\alpha$ -Rényi entropy is Lipschitz continuous, for  $\alpha > 1$ , thus resolving an open problem and providing a substantial improvement over previously known bounds. We also discuss some challenging open questions. This is joint work with Eric Hanson.

**Speaker: Omar Fawzi**

**Title: Variational expressions for quantum divergences and applications**

**Abstract:** For this talk, a variational expression is a way of writing the divergence between two states as the supremum of functions that depend linearly on the two states. Note that such an expression automatically implies joint convexity. As such, a variational expression can be used to define a quantum divergence. In this talk, I will discuss several quantum divergences that can be defined in this way and their applications.

The first application is for the computation of such divergences in a device-independent setting. The second is to establish convergence speed bounds for the regularization of the sandwiched quantum Rényi divergence between quantum channels. The third is to prove a chain rule for the sandwiched quantum Rényi divergence.

This talk is based on joint works with Peter Brown and Hamza Fawzi <https://arxiv.org/abs/2106.13692> and <https://arxiv.org/abs/2007.12576>.

**Speaker: Richard Kueng**

**Title: Complexity by design: k-designs as models for quantum complexity growth**

**Abstract:** The quantum complexity of a unitary transformation or quantum state is defined as the size of the shortest quantum computation that executes the unitary or prepares the state. It is reasonable to expect that the complexity of a quantum state governed by a chaotic many-body Hamiltonian grows linearly with time for a time that is exponential in the system size; however, because it is hard to rule out a shortcut that improves the efficiency of a computation, it is notoriously difficult to derive lower bounds on quantum complexity for particular unitaries or states without making additional assumptions. To go further, one may study more generic models of complexity growth. We provide a rigorous connection between complexity growth and unitary k-designs, ensembles that capture the randomness of the unitary group. This connection allows us to leverage existing results about design growth to draw conclusions about the growth of complexity.

This is joint work with Nicholas Hunter Jones, Wissam Chemissany, Fernando Brandao and John Preskill

**Speaker: Ludovico Lami**

**Title: No second law of entanglement manipulation after all**

**Abstract:** We prove that the theory of entanglement manipulation is asymptotically irreversible under all non-entangling operations, showing from first principles that reversible entanglement transformations require the generation of entanglement in the process. Entanglement is thus shown to be the first example of a quantum resource that does not become reversible under the maximal set of free operations, that is, under all resource non-generating maps. Our result stands in stark contrast with the reversibility of quantum and classical thermodynamics, and implies that no direct counterpart to the second law of thermodynamics can be established for entanglement — in other words, there exists no unique measure of entanglement governing all axiomatically possible state-to-state transformations. This completes the solution of a long-standing open problem [Problem 20 in arXiv:quant-ph/0504166]. We strengthen the result further to show that reversible entanglement manipulation requires the creation of exponentially large amounts of entanglement according to monotones such as the

negativity or the standard robustness of entanglement. Our findings can also be extended to the setting of point-to-point quantum communication, where we show that there exist channels whose parallel simulation entanglement cost exceeds their quantum capacity, even under the most general quantum processes that preserve entanglement-breaking channels. The main technical tool we introduce is the tempered logarithmic negativity, a single-letter lower bound on the entanglement cost that can be efficiently computed via a semi-definite program.

**Speaker: Bartosz Regula**

**Title: Functional analytic insights into irreversibility of quantum resources**

**Abstract:** We propose an approach to quantum resource manipulation based on the basic observation that quantum channels which preserve certain sets of states are contractive with respect to the base norms induced by those sets. In particular, we forgo the usual physical assumptions on quantum dynamics: instead of enforcing complete positivity, trace preservation, or resource-theoretic considerations, we study transformation protocols as norm-contractive maps. This allows us to apply to this problem a technical toolset from functional analysis, unifying previous approaches and introducing new families of bounds for the distillable resources and the resource cost, either one-shot or asymptotic. Since our expressions lend themselves naturally to single-letter forms, they can often be calculated in practice; by doing so, we demonstrate with examples that they can yield the best known bounds on quantities such as the entanglement cost. As applications, we not only give an alternative derivation of the recent result of arXiv:2111.02438 which showed that entanglement theory is asymptotically irreversible, but also provide the quantities introduced in that work with explicit operational meaning in the context of entanglement distillation through a novel generalised form of hypothesis testing relative entropy. Beyond entanglement theory, we reveal a new irreversible quantum resource: through improved bounds for state transformations in the resource theory of magic-state quantum computation, we show that there exist qutrit magic states that cannot be reversibly interconverted under stabiliser protocols.

**Speaker: Daniel Stilck Franca**

**Title: Tomography of many-body quantum states from a few copies through optimal transport**

**Abstract:** We identify conditions on many-body quantum states under which it is possible to infer the expectation values of all quasi-local observables of a given locality up to a relative error from a number of samples that grows polylogarithmically with the system's size and polynomially on the locality of the target observables. This constitutes an exponential improvement over known tomography methods in some regimes. We achieve our results by combining one of the most well-established techniques to learn quantum states, namely the maximum entropy method, with tools from the emerging field of quantum optimal transport. Besides proving these results, we are going to give a pedagogical overview of the underlying technical tools, such as transport-entropy inequalities. This is based on joint work with Cambyse Rouze.

**Speaker: Ivan Todorov**

**Title: An operator algebraic approach to quantum non-local games**

**Abstract:** In this talk, I will outline an operator algebraic view on non-local games with quantum inputs and quantum outputs and their strategies of different types. Restricting to games that obey quantum synchronicity, I will show how their perfect strategies can be described via traces on canonical  $C^*$ -algebras arising from block operator isometries. As an example, I will discuss a non-local game approach to quantum homomorphisms and quantum isomorphisms of non-commutative graphs. The talk will be based on joint papers with L. Turowska and with M. Brannan, S. Harris and L. Turowska.

**Speaker: Marco Tomamichel**

**Title: Catalytic State Transformations in Quantum Resource Theories**

**Abstract:** I will give an overview over catalytic transformations in quantum resource theories, mainly in the resource theory of athermality and entanglement. My talk is going to be based on our recent paper, arXiv:2111.13356, but I will also want to touch on various open problems, small and large.

**Speaker: Péter Vrana**

**Title: Asymptotic equipartition property for a source having ambiguous alphabet**

**Abstract:** We propose a generalization of the asymptotic equipartition property to discrete sources with an ambiguous alphabet, and prove that it holds for irreducible stationary Markov sources with an arbitrary distinguishability relation. Our definition is based on the limiting behavior of certain graph parameters, evaluated at subgraphs of strong powers of the confusability graph induced on high-probability subsets. The results provide an information-theoretic interpretation of the graph entropy rate of such sources.

# Posters

**Presenter:** Gergely Bunth

**Title:** Equivariant relative submajorization

**Abstract:** A pair equivariantly relatively submajorizes another if there is an equivariant subnormalized channel that takes the components of the first pair to the second pair satisfying similar positivity constraints as in the definition of relative submajorization. We find a sufficient condition for the existence of catalytic transformations and a characterization of an asymptotic relaxation of the relation. For classical and certain quantum pairs the characterization is in terms of explicit monotone quantities related to the sandwiched quantum Rényi divergences. In the general quantum case the relevant quantities are given only implicitly. Nevertheless, with the use of matrix means, we find a large collection of monotones that provide necessary conditions for asymptotic or catalytic transformations. When applied to time-translation symmetric maps, these give rise to second laws that constrain state transformations allowed by thermal operations even in the presence of catalysts.

**Presenter:** Máté Farkas

**Title:** Mutually unbiased measurements

**Abstract:** Mutually unbiased bases (MUBs) are widely useful measurements in quantum information, used in state determination, quantum cryptography, quantum communication and other tasks. In this work we introduce a generalisation of MUBs that we call mutually unbiased measurements (MUMs). MUMs are defined through a complementarity property: if a measurement yields a definite outcome on a quantum state then a measurement unbiased to it yields a uniformly random outcome on the same state. MUMs are the same as MUBs whenever the Hilbert space dimension matches the outcome number. In general, MUMs admit the same incompatibility robustness and the same entropic uncertainty relations as MUBs. We characterise MUMs via block matrices that we call Hadamard matrices of unitaries. We show that a pair of MUMs is a direct sum of MUBs if and only if all blocks of the corresponding Hadamard matrix commute. Using this characterisation, we show that there exist MUMs that are not direct sums of MUBs, and we give explicit constructions through a correspondence with quaternionic Hadamard matrices. We further show that there exist MUMs that cannot even be mapped to MUBs via any completely positive unital map.

We introduce a family of Bell inequalities whose maximal violation certify precisely the MUM property. Due to the fact that there exist unitarily inequivalent MUBs, this result also implies that the quantum correlation maximally violating these Bell inequalities is in general an extremal point but not a self-test, the first example of such a correlation. We further show that the maximal violation certifies  $\log(d)$  bits of device-independent secret key. Then, we generalise the inequalities to an arbitrary number of MUMs (instead of two) and by numerically optimising these inequalities in a fixed dimension we tackle the long-standing problem of the number of MUBs in composite dimensions, known as Zauner's conjecture.

Based on: Science Advances 7 eabc3847, arXiv:2204.11886, arXiv:2203.09429

**Presenter: Péter Frenkel**

**Title: Classical simulations of communication channels**

**Abstract:** We investigate whether certain non-classical communication channels can be simulated by a classical channel with a given number of states and a given ‘amount’ of noise. The main result is that noisy quantum channels can be simulated by a corresponding classical channel with ‘the same amount’ of noise, in a suitable sense. Classical simulations of general probabilistic channels are also studied.

**Presenter: Felix Huber**

**Title: Dimension-free entanglement detection in multipartite Werner states**

**Abstract:** Werner states are multipartite quantum states that are invariant under the diagonal conjugate action of the unitary group. This paper gives a complete characterization of their entanglement that is independent of the underlying local Hilbert space: for every entangled Werner state there exists a dimension-free entanglement witness. The construction of such a witness is formulated as an optimization problem. To solve it, two semidefinite programming hierarchies are introduced. The first one is derived using real algebraic geometry applied to positive polynomials in the entries of a Gram matrix, and is complete in the sense that for every entangled Werner state it converges to a witness. The second one is based on a sum-of-squares certificate for the positivity of trace polynomials in noncommuting variables, and is a relaxation that involves smaller semidefinite constraints.

Joint work with Igor Klep, Victor Magron, Jurij Volčič.

**Presenter: Baksa Kolok and Tamás Tasnádi**

**Title: Geometric aspects of classical and quantum convertibility**

**Abstract:** Convertibility questions have become increasingly relevant since the turn of the millennium in many quantum resource theories, such as quantum thermodynamics and LOCC theory. The main idea of this work thesis is to associate certain geometric objects (Lorenz zonotopes and measurement polytopes) to  $n$ -tuples of (classical or quantum) states and investigate the connections between the inclusion relation of these geometric objects and the algebraic relations of convertibility. Generally the algebraic (convertibility) relations imply the geometric (inclusion) relations but the converse direction does not always hold.

Joint work with Tamás Tasnádi

**Presenter: Gábor Maróti**

**Title: Super-exponential distinguishability of correlated quantum states**

**Abstract:** In the problem of asymptotic binary i.i.d. state discrimination, the optimal asymptotics of the type I and the type II error probabilities is in general an exponential decrease to zero as a function of the number of samples; the set of achievable exponent pairs is characterized by the quantum Hoeffding bound theorem. A super-exponential decrease for both types of error probabilities is only possible in the trivial case when the two states are orthogonal, and hence can be perfectly distinguished using only a single copy of the system.

Here we show that a qualitatively different behaviour can occur when there is correlation between the samples. Namely, we use gauge-invariant and translation-invariant quasi-free

states on the algebra of the canonical anti-commutation relations to exhibit pairs of states on an infinite spin chain with the properties that a) all finite-size restrictions of the states have invertible density operators, and b) the type I and the type II error probabilities both decrease to zero at least with the speed  $e^{-nc \log n}$  with some positive constant  $c$ , i.e., with a super-exponential speed in the sample size  $n$ .

Based on arXiv:2203.16511, joint work with Gergely Buntth, Milán Mosonyi and Zoltán Zimborás.

**Presenter: Zsombor Szilágyi**

**Title: On the error exponents of binary state discrimination with composite hypotheses**

**Abstract:** The trade-off between the two types of error probabilities in binary state discrimination may be quantified in the asymptotics by various error exponents. In the composite case, where the hypotheses consist of sets of states, any such exponent is upper bounded by the infimum of the corresponding pairwise exponents of discriminating individual members of the two sets. Attainability of this upper bound may depend on the type of exponents considered; whether the problem is classical or quantum; the cardinality and the geometric properties of the sets representing the hypotheses; and also on the dimensionality of the underlying Hilbert space. Our main contribution is clarifying this landscape considerably. We show that in the quantum case unattainability is the general behaviour for all the exponents, already in finite dimension, for a simple null-hypothesis and an alternative hypothesis consisting of only two states. Moreover, the upper bound may be strict even in the infinite-dimensional classical case if the alternative hypothesis contains at least countably infinitely many states. We also prove general attainability results, e.g., for classical adversarial and arbitrarily varying hypothesis testing, and for the quantum case where all states are pure, or the states commute with every state in the opposite hypothesis.

Based on joint work with Milán Mosonyi and Mihály Weiner, arXiv:2011.04645.