The Boltzmann-Sinai Ergodic Hypothesis

Definition: Hard Ball Systems

Consider a system of $N \geq 2$ hard balls of masses $m_1, \ldots, m_N > 0$ and of radii r in the $\nu \geq 2$ unit torus. The dynamics is the usual: uniform motion with elastic collisions of the balls. Invariants of the motion are the total energy (assume it is equal to 1), the total momentum (assume that it is equal to 0). The center of mass, in general, does not exist but, nevertheless one can factorize the system by joint shifts of the ball configuration . It is easy to see that by fixing these invariants and by factorizing as said above the system will be isomorphic to a cylindric billiard. The cylinders correspond to pairs of balls, their bases are ν-dimensional balls. (The construction is described in detail in N. Simányi-D. Szász: Hard Ball systems are Completely Hyperbolic. *Annals of Mathematics*, 149 (1999), 35-96.)

The Boltzmann-Sinai Ergodic Hypothesis: *For any $N, \nu; m_1, \ldots, m_N; r$, the hard ball system is ergodic.*

For typical systems, i. e. for any N, ν and for almost every values of $m_1, \ldots, m_N; r$ the hypothesis is proved in a recent manuscript of N. Simányi: Proof of the Ergodic Hypothesis for Typical Hard Ball Systems, preprint, October 2002.

Thus the main remaining questions are:

Problem: *Prove the hypothesis for every value of the parameters.* (In particular, prove it for the case when all masses are equal.)

Remark Hard ball systems are particular cases of cylindric billiards and consequently the Boltzmann-Sinai Ergodic Hypothesis for hard ball systems is weaker that the Erdőtarcsa Conjecture.

Discussion: The appearance of exceptional parameters comes from the algebraic method of the aforementioned work of Simányi and Szász. Indeed, according to Simányi's most recent result in dimension $\nu = 2$ the ergodic hypothesis is true for ALL values of the parameters $N, m_1, \ldots, m_N; r$.

By general results, if once these systems are ergodic, then they are Kolmogorov mixing, and further Bernoulli systems as well.