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REPRESENTATION THEORY/SPRING 2010/ E. Horváth and A. Küronya
PRACTICE SESSION 9

1. a) Let L be a Lie algebra and $K \leq L$ a subalgebra. Prove that the **normalizer** $N_L(K) := \{x \in L | [x, K] \subseteq K\}$ is a subalgebra in L.
b) Prove that $K \lhd N_L(K)$, and $N_L(K)$ is the biggest subalgebra in L, where K is an ideal.

2.a) Prove that the **centralizer** $C_L(X) := \{a \in L | [a, X] = 0\}$ of a subset X of the Lie algebra L is a subalgebra in L.
b) Prove that $C_L(L) = Z(L)$.
c) Prove that if $K \leq L$, $C_L(K) \lhd N_L(K)$.

3.a)Let $ad : L \to gl(L)$ be the adjoint representation of the Lie algebra L. Prove that $Ker(ad) = Z(L)$.
b) Prove that if L is simple then ad is injective, hence every simple Lie algebra is isomorphic to a linear Lie algebra.

4. Prove that if $\delta : L \to L$ is a derivation then $\delta^n([x, y]) = \sum_{i=0}^{n} \binom{n}{i} [\delta^i(x), \delta^{n-i}y]$.

5.a) Prove that the automorphisms $Aut(L)$ of a Lie algebra is a group.
b)(HW) Prove that if $\psi \in Aut(L)$ then $\psi(ad^i x)\psi^{-1} = ad^i(\psi(x))$
c)Prove that the subgroup of inner automorphisms $Int(L)$, namely the subgroup generated by those automophisms of L which have the form $\delta = exp(adx)$, where $adx : L \to L$ is a nilpotent linear transformation, is a normal subgroup in $Aut(L)$.

6. Prove that if L^i is the ith member of the lower central series of the Lie algebra L then $[L^i, L^j] \leq L^{i+j}$.

7. Prove that $t(n, F)$ is a solvable Lie algebra, but it is not nilpotent.

8. Prove that $L^{(i)} \leq L^i$.

9. Prove that $n(n, F)$ is a nilpotent Lie algebra.

10.(HW) Prove that the normalizer of $t(n, F)$ in $gl(n, F)$ is $t(n, F)$, the normalizer of $d(n, F)$ in $gl(n, F)$ is $d(n, F)$ and the normalizer of $n(n, F)$ in $gl(n, F)$ is $t(n, F)$.

11. Prove that if $charF = 2$, then $sl(2, F)$ is nilpotent.