Topology A (TOPA) / Alex Küronya / Fall 2008

Homework 9

Due date: May 15$^{\text{th}}$

The problems with an asterisk are the ones that you are supposed to submit. The ones with two asterisks are meant as more challenging, and by solving them you can earn extra credit.

1. Let X,Y be topological spaces, Y discrete. Verify that the projection map $\pi : X \times Y \to X$ is a covering map.

2. * Consider a covering map $p : E \to B$ with B connected. Prove that if for some $x \in B$ the fibre $p^{-1}(x) \subseteq E$ has m elements, then $p^{-1}(b)$ has m elements for every $b \in B$. (In this case the covering map p is called an *m-fold covering* of B.)

3. If $p : X \to Y$ and $q : Y \to Z$ are covering maps, and $(q \circ p)^{-1}(z)$ is finite for every $z \in Z$, then $q \circ p$ is also a covering map.

4. Let $p : E \to B$ be a covering map with E path-connected. Show that if B is simply connected, then p is a homeomorphism.

5. Let $p : E \to B$ be a covering map. Show that if B is Hausdorff/regular, then so is E.

6. * If X,Y are topological spaces, $x_0 \in X$, $y_0 \in Y$, then prove that

$$\pi_1(X \times Y, (x_0, y_0)) \simeq \pi_1(X, x_0) \times \pi_0(Y, y_0) \ .$$

7. Show that a closed continuous surjective map is a quotient map.