Topology (TOP) / Alex Küronya / Spring 2009

Homework 10

Due date: May 22th

The problems with an asterisk are the ones that you are supposed to submit. The ones with two asterisks are meant as more challenging, and by solving them you can earn extra credit.

1. * Let $f : \mathbb{S}^1 \to X$ be a nullhomotopic map. Show that f extends to a continuous map $\tilde{f} : \mathbb{D}^2 \to X$.

2. For a retract $A \subset \mathbb{D}^2$, prove that every continuous map $f : A \to A$ has a fixed point.

3. Let $f : \mathbb{S}^2 \to \mathbb{S}^2$ be a continuous map such that $f(x) \neq f(-x)$ for every $x \in \mathbb{S}^2$. Show that g is surjective.

4. * Compute the fundamental group of the real projective plane $\mathbb{RP}^2$.

5. Determine the fundamental group of $\mathbb{S}^1 \times \mathbb{D}^2$ and of $\mathbb{S}^1 \times \mathbb{S}^2$.

6. Accepting the fact that for an arbitrary positive integer n, no antipode preseving map $f : \mathbb{S}^n \to \mathbb{S}^n$ is nullhomotopic, prove the following statements:
   (1) There exists no retraction $r : \mathbb{D}^{n+1} \to \mathbb{S}^n$.
   (2) There exists no antipode-preserving map $g : \mathbb{S}^{n+1} \to \mathbb{S}^n$.

7. Using the method for determining the fundamental group of the circle, prove that
$$\pi_1(T^2) \simeq \mathbb{Z} \times \mathbb{Z} \ .$$

8. Prove that a nonsingular 3×3 matrix with nonnegative real entries has a positive real eigenvalue.

9. Consider the polynomial equation $x^n + a_{n-1}x^{n-1} + \cdots + a_0 = 0$ with complex numbers as coefficients. Verify that if
$$\sum_{i=0}^{n-1} |a_i| < 1$$
then all roots lie in the open unit ball $\mathbb{B}(0, 1)$.