Topology (TOP) / Alex Küronya / Spring 2006

Homework 10

The problems with an asterisk are the ones that you are supposed to submit. The ones with two asterisks are meant as more challenging, and by solving them you can earn extra credit.

1. * Let $f : \mathbb{S}^1 \to X$ be a nullhomotopic map. Show that $f$ extends to a continuous map $\tilde{f} : \mathbb{D}^2 \to X$.

2. For an arbitrary subset $A \subset \mathbb{D}^2$, prove that every continuous map $f : A \to A$ has a fixed point.

3. Let $f : \mathbb{S}^2 \to \mathbb{S}^2$ be a continuous map such that $f(x) \neq f(-x)$ for every $x \in \mathbb{S}^2$. Show that $g$ is surjective.

4. If $X$,$Y$ are topological spaces, $x_0 \in X$, $y_0 \in Y$. Show that

$$\pi_1(X \times Y, (x_0, y_0)) \simeq \pi_1(X, x_0) \times \pi_0(Y, y_0) \ .$$

5. * Compute the fundamental group of the real projective plane $\mathbb{RP}^2$.

6. Determine the fundamental groups of $\mathbb{S}^1 \times \mathbb{D}^2$ and $\mathbb{S}^1 \times \mathbb{S}^2$.

7. Accept the fact that for an arbitrary positive integer $n$, no antipode preseving map $f : \mathbb{S}^n \to \mathbb{S}^n$ is nullhomotopic. Using this result, prove the following statements:

(1) There exists no retraction $r : \mathbb{D}^{n+1} \to \mathbb{S}^n$.
(2) There exists no antipode-preserving map $g : \mathbb{S}^{n+1} \to \mathbb{S}^n$.

8. ** Let $Y$ be a deformation retract of the topological space $X$, let $a \in A$. Show that the inclusion map $j : (A, a) \to (X, a)$ induces an isomorphism of the fundamental groups.