Topology (TOP) / Alex Küronya / Fall 2005

Final exam

1. Prove that if $f : X \to Y$ and $g : Z \to W$ are open identification maps between topological spaces, then so is $f \times g : X \times Z \to Y \times W$.

2. Show that if for a continuous map $f : \mathbb{S}^2 \to \mathbb{S}^2$, $f(x) \neq f(-x)$ holds for every $x \in \mathbb{S}^2$, then f is surjective.

3. Let X, Y, Z be topological spaces, $f : X \times Y \to Z$ be an arbitrary function. Prove that f is continuous if and only if it is continuous in both variables separately.

4. Let (X, d) be a compact metric space, $\mathfrak{U}$ an open covering of X. Prove that there exists a positive real number $\delta > 0$ such that for every $A \subseteq X$ with diameter less than δ there exists an open set $U \in \mathfrak{U}$ such that $A \subseteq U$.

5. Determine the fundamental group of the space $\mathbb{S}^1 \times \mathbb{D}^2$. Justify your answer.

6. Let $f : X \to Y$ be a perfect closed continuous surjective map. Verify that if X is Hausdorff, then so is Y.

Bonus problem:

7. Let X be a compact Hausdorff topological space; $\{Y_n\}$ a countable collection of closed subsets of X, such that the interior of each Y_n in X is empty. Show that

$$\operatorname{int} \bigcup_{n=1}^{\infty} Y_n = \emptyset$$

as well.

Have fun!