Topology A (TOPA) / Alex Küronya / Fall 2009

Review problems

Consider the following topological spaces.

(1) $X$ arbitrary set, $\tau = \{\emptyset, X\}$
(2) $X$ arbitrary set, $\tau = 2^X$
(3) $X$ arbitrary set, $\tau = \{U \subseteq X \mid X \setminus U \text{ is finite }\} \cup \{\emptyset\}$
(4) $X$ arbitrary set, $\tau = \{U \subseteq X \mid X \setminus U \text{ is countable }\} \cup \{\emptyset\}$
(5) $X = \mathbb{R}$, $\tau =$ the topology generated by the intervals $[x, y)$ and $(x, y]$ for all $x, y \in \mathbb{R}$
(6) $X = [0, 1]$, $\tau =$ the topology whose closed subsets are $\left\{[0, \frac{1}{n}] \mid n \in \mathbb{N}^{>0}\right\} \cup \{\{0\}\} \cup \{\emptyset\}$
(7) $X = \mathbb{N}$, $\tau = \{[n, \infty) \mid n \in \mathbb{N}\} \cup \{\emptyset\}$
(8) $X = \mathbb{N}$, $\tau =$ the topology whose closed subsets are $\{[n, \infty) \mid n \in \mathbb{N}\} \cup \{\emptyset\}$
(9) $X = \mathbb{R}$, $\tau =$ the classical topology
(10) $X = \mathbb{Q}$, $\tau =$ the classical topology
(11) $X = [0, 1]$, $\tau =$ the classical topology
(12) $X = (0, 1)$, $\tau =$ the classcial topology
(13) $X = (X_1, \tau_1) \times (X_2, \tau_2)$ where $X_1$ and $X_2$ are arbitrary sets, and both $\tau_1$ and $\tau_2$ have the cofinite topology.
(14) $[0, 1] \times (0, 1)$, $\tau =$ the product of the classical topology on both factors.
(15) $X = \mathbb{R}^2 \setminus \{(0, 0)\}$, $\tau =$ the classical topology.

1. In each case, check whether the given collection of subsets is indeed a topology.

2. For each topological space $(X, \tau)$ on the list decide, decide whether it is
   (1) connected
   (2) path-connected
   (3) totally disconnected
   (4) discrete
   (5) compact
   (6) irreducible
   (7) locally connected

3. For every topological space above, find the largest $i$ for which $(X, \tau)$ is $T_i$.