Topology (TOP) / Alex Küronya / Fall 2006

Homework 8

The problems with an asterisk are the ones that you are supposed to submit. The ones with two asterisks are meant as more challenging, and by solving them you can earn extra credit.

1. Is it true that all convex subsets of $\mathbb{R}^n$ (equipped with the Euclidean topology) are contractible?

2. * Show that a retract of a contractible topological space is contractible.

3. ** Prove that for any pair of maps $f, g : \mathbb{S}^n \to \mathbb{S}^n$ with $\forall x \in \mathbb{S}^n$ $f(x) \neq -g(x)$, one has $f \simeq g$.

4. * Let $f : X \to Y$ be homotopic maps, with f being a homeomorphism. Does it follow that g is a homeommorphism as well?

5. Which of the following topological properties are preserved under homotopy equivalence of topological spaces (i.e. if $X \simeq Y$ and X has the property in question then does Y have it automatically?): compactness, connectedness, path-connectedness, first countability, second countability, T_0, T_1, T_2, T_3, T_4?

6. Which of the following properties are preserved under homotopy of maps: injective, surjective, open, closed, perfect, proper?