Topology (TOP) / Alex Küronya / Fall 2005

Homework 8

The problems with an asterisk are the ones that you are supposed to submit. The ones with two asterisks are meant as more challenging, and by solving them you can earn extra credit.

1. Let X, Y be topological space, $A \subseteq X$ a closed subset, $f : A \to Y$ a map. Consider the quotient space $Y \cup_f X$. Show that the natural inclusion $Y \hookrightarrow Y \cup_f X$ maps Y onto a closed subspace, while the image of the inclusion $X - A \hookrightarrow Y \cup_f X$ is open.

2. Prove that if $f : X \to Y$, $g : Z \to W$ are open identification maps, then so is $f \times g : X \times Z \to Y \times W$, where $(f \times g)(x, z) \stackrel{\text{def}}{=} (f(x), g(z))$.

3. Is it true that all convex subsets of $\mathbb{R}^n$ (equipped with the Euclidean topology) are contractible?

4.* Let X, Y be normal topological spaces, $A \subseteq X$ a closed subset with a map $f : A \to Y$. Verify that $Y \cup_f X$ is normal as well.

5. * Show that the retract of a contractible topological space is contractible.

6. ** Prove that for any pair of maps $f, g : \mathbb{S}^n \to \mathbb{S}^n$ with $\forall x \in \mathbb{S}^n\ f(x) \neq -g(x)$, one has $f \simeq g$.

7. We call a subset X in a topological group G *symmetric*, if $X^{-1} = X$. Show that the symmetric neighbourhoods of the identity element 1_G of G form a neighbourhood basis of 1_G.

8 ** Let G be a topological group, $H \subseteq G$ a closed subgroup. Prove that the space

$$G/H \stackrel{\text{def}}{=} \{\, gH \mid g \in G \,\}$$

of left cosets of H equipped with the quotient topology induced by the canonical projection $\pi : G \to G/H$ is a Hausdorff topological space. Verify in addition that the map π is open.