Topology (TOP) / Alex Küronya / Fall 2005

Homework 10

The problems with an asterisk are the ones that you are supposed to submit. The ones with two asterisks are meant as more challenging, and by solving them you can earn extra credit.

1. * Show that if X and Y and path-connected topological spaces, then

$$\pi_1(X \times Y) \simeq \pi_1(X) \times \pi_1(Y) \ .$$

2. Prove that if $\mathbb{S}^2 = A_1 \cup A_2 \cup A_3$ with each $A_i \subseteq \mathbb{S}^2$ closed, then at least one of these sets must contain a pair of antipodal points.

3. * Verify that for any paths f_0, f_1, g_1, g_1 in a topological space X, $f_0 * g_0 \simeq f_1 * g_1$ and $g_0 \simeq g_1$ together imply $f_0 \simeq f_1$.

4. ** Let X be a path-connected topological space. Show that $\pi_1(X)$ is abelian if and only if all basepoint-change homomorphisms τ_h depend only on the endpoints of h.

5. Prove that if for a continuous map $f : \mathbb{S}^2 \to \mathbb{S}^2$, $f(x) \neq f(-x)$ holds for every $x \in \mathbb{S}^2$, then f is surjective.

6. Find a circle about the origin containing all the roots of the polynomial $x^{17} + x^2 - 1$.