Name: Neptun:

A3 Sample Final Exam, Fall 2018/19
90 minutes. Each problem is worth 15 points.

Minimum requirement: get at least 15 points from the first three problems, otherwise the remaining problems are not corrected. Passing condition: reach at least 48 points (40%) in the final exam and altogether with your midterm tests at least 100 points (out of 200).

1. Solve the following system of differential equations:

$$x' + 2y' - 3x + 4y = 2\sin t, \qquad 2x' + y' + 2x - y = \cos t.$$

2. The distribution of X is given by the density function

$$f(x) = 2x, \quad \text{if } 0 < x < 1, \quad \text{and } f(x) = 0 \text{ otherwise.}$$

   Determine the probability $\mathbf{P}(0.25 < X < 0.5)$, then find the expectation of X!

3. Solve the following second order differential equation with missing term

$$xy'' - y' = x^3.$$

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4. Solve the following initial value problem:

$$y'' + y' - 12y = 0, \quad y(2) = 2,\ y'(2) = 0.$$

5. Find the equilibrium solutions of the autonomous differential equation

$$y' = y(y^2 + 7y - 30)$$

   and characterize them from the point of view of stability.

6. Consider an experiment that consists of counting the number of α-particles given off in a 1-second interval by 1 gram of radioactive material. If we know from past experience that, on the average, 3.2 such α-particles are given off, what is a good approximation to the probability that no more than 2 α-particles will appear? What if we look at a 2-second interval?

7. Let A and B be independent events with respective probabilities 0.4 and 0.7. Find the probability that exactly one of them happens

8. There are 100 students registered for an overall course, but each of them attends the lectures with probability 0.8, independently. What size of a class (with how many chairs) to reserve if we want to give only 5 percent chance to the situation that a student, arriving to the class, cannot find a chair to sit on.