1st Olympiad of Metropolises
Mathematics · Day 2

Problem 4. A convex quadrilateral $ABCD$ has right angles at A and C. A point E lies on the extension of the side AD beyond D so that $\angle ABE = \angle ADC$. The point K is symmetric to the point C with respect to point A. Prove that $\angle ADB = \angle AKE$.

Problem 5. Let $r(x)$ be a polynomial of odd degree with real coefficients. Prove that there exist only finitely many (or none at all) pairs of polynomials $p(x)$ and $q(x)$ with real coefficients satisfying the equation $(p(x))^3 + q(x^2) = r(x)$.

Problem 6. In a country with n cities, some pairs of cities are connected by one-way flights operated by one of two companies A and B. Two cities can be connected by more than one flight in either direction. An AB-word w is called *implementable* if there is a sequence of connected flights whose companies' names form the word w. Given that every AB-word of length 2^n is implementable, prove that every finite AB-word is implementable. (An AB-word of length k is an arbitrary sequence of k letters A or B; e. g. $AABA$ is a word of length 4.)