1st Olympiad of Metropolises

Mathematics · Day 1

Problem 1. Find all positive integers n such that there exist n consecutive positive integers whose sum is a perfect square.

Problem 2. Let $a_1, \dots, a_n$ be positive integers satisfying the inequality

$$\sum_{i=1}^{n} \frac{1}{a_i} \leq \frac{1}{2}\,.$$

Every year, the government of Optimistica publishes its *Annual Report* with n economic indicators. For each $i = 1, \dots, n$, the possible values of the i-th indicator are $1, 2, \dots, a_i$. The Annual Report is said to be *optimistic* if at least $n-1$ indicators have higher values than in the previous report. Prove that the government can publish optimistic Annual Reports in an infinitely long sequence.

Problem 3. Let $A_1A_2\dots A_n$ be a cyclic convex polygon whose circumcenter is strictly in its interior. Let B_1, B_2, ..., B_n be arbitrary points on the sides A_1A_2, A_2A_3, ..., A_nA_1, respectively, other than the vertices. Prove that

$$\frac{B_1B_2}{A_1A_3} + \frac{B_2B_3}{A_2A_4} + \dots + \frac{B_nB_1}{A_nA_2} > 1\,.$$