Large Scale Behaviour of Interacting Particle Systems: Fluctuations and Hydrodynamics

Budapest, 22-26 August 2005

Organized within the framework of the project

Phase Transitions and Fluctuation Phenomena for Random Dynamics in Spatially Extended Systems (RDSES)

of the European Science Foundation.

http://www.esf.org/rdses

ABSTRACTS OF TALKS:

Márton Balázs (University of Wisconsin):

Construction of the zero range process and a deposition model with superlinear growth rates

(Joint work with Firas Rassoul-Agha and Timo Seppäläinen)

Construction of the zero range process and some related deposition processes is available following the methods initiated by T. M. Liggett and F. Spitzer, and E. D. Andjel. These methods work under the assumption that the jump rates of the process are either bounded, or at least are of bounded increments as functions of the local configuration. However, recent results involving stationary shock measures shed light on processes with jump rates depending *exponentially* on the local configuration. The methods mentioned above do not seem to extend for constructing the dynamics of such a process. The talk will sketch estimates based on probabilistic coupling arguments, which make a construction possible in the case of (at most) exponentially growing jump rates. Unfortunately, difficulties arising due to the fast-growing rates only allowed us to establish parts of the usual Markov-semigroup machinery.

Gérard Ben Arous (NYU Courant Institute):

Title to be announced

Abstract not available.

Jacob van den Berg (CWI Amsterdam):

Conditional correlation inequalities in static and dynamic contexts

(Joint work with J. Kahn and O. Haggstrom)

Consider ordinary bond percolation on a finite or countably infinite graph. Let s, t, a and b be vertices. In a paper with Kahn (2001) it has been shown that, conditioned on the event that there is no open path from s to t, the two events "there is an open path from s to a" and "there is an open path from s to b" are positively correlated. In recent papers with Haggstrom and Kahn this result has been extensively generalized, with applications to contact processes. Preservation (in a temporal as well as in spatial sense) has also been investigated. A dual version of our results gave a proof of a conjecture of N. Konno for the 1D contact process. In this talk I will explain these results and discuss relations with results by Belitsky, Ferrari, Konno and Liggett (1997), Liggett and Steif (2004) and Liggett (2005).

Lorenzo Bertini (Univ. La Sapienza, Roma):
Title to be announced

Abstract not available.

Erwin Bolthausen (Universität Zürich):
Exit distribution for random walks in random environments
(Joint work with Ofer Zeitouni)

We consider the standard model for random walks on $\mathbb{Z}^d$ with i.i.d. random environments. We prove that if the disorder is small, and $d \geq 3$, then the exit distributions from large boxes are close to the exit distributions of the standard random walk. The distance is measured in terms of the total variation of slightly smoothed distributions. The proof is given by a rigorous version of renormalization. It is more elementary than the approach used by Bricmont and Kupiainen [1]. For recent results on (continuous time) diffusions in random environments, see also [2].

References

[1] Bricmont, J., and Kupiainen, A.: *Random walks in asymmetric random environments.* Comm. Math. Phys. **142**, 345-420 (1991)

[2] Sznitman, A.-S., and Zeitouni, O.: *An invariance principle for isotropic diffusions in random environment.* Preprint

Maury Bramson (University of Minnesota):
Exclusion Processes in Higher Dimensions
(Joint work with Thomas Liggett)

Exclusion processes form one of the major classes of interacting particle systems. There, particles on a lattice execute independent random walks in continuous time, except when the target site is already occupied, in which case the particle remains at the original site. Many results exist for the lattice $\mathbb{Z}$. In particular, the equilibria of such exclusion processes are in many cases well understood. Little is presently known, however, for $\mathbb{Z}^d$, $d > 1$. We will review the behavior of the exclusion process on $\mathbb{Z}$ and present the foundation of a theory for $d > 1$. We will conclude with various open problems.

Francis Comets (Université Paris 7):

On multidimensional branching random walks in random environment

(Joint work with Serguei Popov)

We study branching random walks in random i.i.d. environment in $\mathbb{Z}^d, d \geq 1$. For our model, the population size cannot decrease, and a natural definition of recurrence is introduced. We prove a dichotomy for recurrence/transience, depending only on the support of the environmental law. We give sufficient conditions for recurrence and for transience. We then focus on the recurrent case: we study the tails of the hitting times, and we prove a shape theorem for the set of lattice sites which are visited up to a large time. We also investigate the connections with random walks in random environment.

Jean-Dominique Deuschel (TU Berlin):

Aging for interacting diffusion processes

(Joint work with Amir Dembo)

We study the aging phenomena for a class of interacting diffusion processes $\{X_t(i), i \in \mathbb{Z}^d\}$. In this framework we see the effect of the lattice dimension d on aging, as well as that of the class of test functions $f(X_t)$ considered. We further note the sensitivity of aging to specific details, when degenerate diffusions (such as super random walk, or parabolic Anderson model), are considered. We complement our study of systems on the infinite lattice, with that of their restriction to finite boxes. In the latter setting we consider different regimes in terms of box size scaling with time, as well as the effect that the choice of boundary conditions has on aging. The key tool for our analysis is the random walk representation for such diffusions.

Tadahisa Funaki (University of Tokyo):

Motion of Winterbottom-like droplets in SPDEs with smooth noise

We shall discuss the singular limit as $\epsilon \downarrow 0$ for a parabolic stochastic partial differential equation (SPDE) of fourth order:

$$\frac{\partial u}{\partial t} = -\frac{1}{\epsilon}\Delta\{\Delta u + f(u)\} + \text{div } \dot{\mathbf{w}}(t,x), \quad x \in \mathbb{T}^d, \tag{1}$$

on a torus $\mathbb{T}^d$ with a smooth Gaussian noise $\mathbf{w}(t,x) = (w^i(t,x))_{i=1}^d$, and derive a diffusive motion of Winterbottom-like droplets in the limit.

In general, for an interacting system, under the situation that the corresponding hydrodynamic (macroscopic) equation admits a continuous family of stationary solutions, one can expect to observe their fluctuations under a properly chosen time scale beyond the hydrodynamic one. One possible choice for such microscopic system might be the Ginzburg-Landau $\nabla\varphi$-interface model of conservative type with pinning and repulsion,

cf. [2]. In this system, although the hydrodynamic limit is not established at present, the stationary solutions of the corresponding macroscopic equation ought to be the so-called Winterbottom shapes [1] which have the degree of freedom of translation.
Since this model itself has several technical difficulties, we modify it in the following aspects and introduce the SPDE (1):

(a) We start with the macroscopic continuous system on a torus instead of the microscopic lattice system replacing the noise with smooth one.

(b) At the same time, the microscopic self potential describing pinning and repulsion is replaced by the macroscopic one, which is smooth.

In fact, for the $\nabla\varphi$-interface dynamics of conservative type with a quadratic interaction potential made the modification (b) being defined on a lattice torus with side length N, the hydrodynamic time scale is of $O(N^4)$, which leads to the macroscopic equation $\partial u/\partial t = -\Delta\{\Delta u + f(u)\}$ in the limit. Then, choosing the longer time scale of $O(N^{4+d})$, which keeps the noise term of $O(1)$, the SPDE (1) naturally arises with $\epsilon = N^{-d}$.

References

[1] E. Bolthausen and D. Ioffe, *Harmonic crystal on the wall: a microscopic approach*, Commun. Math. Phys., **187** (1997), pp. 523–566.

[2] T. Funaki, *Stochastic Interface Models*, In: Lectures on Probability Theory and Statistics, Ecole d'Eté de Probabilités de Saint-Flour XXXIII - 2003 (ed. J. Picard), Lect. Notes Math., **1869** (2005), Springer.

Stefan Großkinsky (TU München):

Spontaneous symmetry breaking in a two-species driven diffusive system

(Joint work with R.D. Willmann and G.M. Schütz)

We study a one-dimensional driven two-species model with parallel sublattice update and open boundaries. Although the microscopic dynamics is symmetric with respect to the two species and interactions are short-ranged, there is a region in parameter space with broken symmetry in the steady state. The sublattice update is deterministic in the bulk and allows for a detailed analysis of the relaxation dynamics. Symmetry breaking is shown to be the result of an amplification mechanism of fluctuations and in contrast to previously considered models, the proof is valid throughout the whole region in parameter space.

Herve Guiol (IMAG), **Ellen Saada** (Université de Rouen):

Euler hydrodynamics of one-dimensional attractive systems

(Joint work with C. Bahadoran and K. Ravishankar)

We consider asymmetric, attractive, irreducible, conservative particle systems on $\mathbb{Z}$ with at most K particles per site for which no knowledge of explicit invariant measure is assumed. Typical examples of such processes are the simple exclusion and the misanthrope processes (for which the invariant measures are known) and the K-exclusion processes. We will focus on the hydrodynamic under Euler scaling of these systems.

Frank den Hollander (EURANDOM and Leiden University)):

Intermittency on catalysts

(Joint work with Jürgen Gärtner and Gregory Maillard)

In this talk we look at the parabolic Anderson equation

$$\frac{\partial u}{\partial t} = \kappa \Delta u + \xi u, \qquad u|_{t=0} \equiv 1,$$

where $u\colon \mathbb{Z}^d \times [0,\infty) \to \mathbb{R}$, κ is the diffusion constant, Δ is the discrete Laplacian, and $\xi\colon \mathbb{Z}^d \times [0,\infty) \to \mathbb{R}$ is a space-time random medium. The solution of the equation describes the evolution of a "reactant" u under the influence of a "catalyst" ξ.
We consider three choices for ξ: (1) zero-range process, (2) exclusion process, (3) voter model. We study the annealed Lyapunov exponents, i.e., the exponential growth rates of the successive moments of u. We show that these exponents display an interesting dependence on the diffusion constant κ, with qualitatively different behavior in different dimensions.

Milton Jara (IMPA):

Non-equilibrium fluctuations for a simple exclusion process in random environment

Consider a sequence $\{\xi_x\}_x$ of positive, i.i.d. random variables, and fix a realization of the environment ξ. Consider a simple exclusion process in $\mathbb{Z}$ in which each particle jumps from x to $x+1$ or from $x+1$ to x with an exponential rate ξ_x. Under the hypothesis $\gamma = E[\xi_0^{-1}] < +\infty$, we prove that the hydrodynamic limit of this model is given by the heat equation $\partial_t \rho = \gamma^{-1} \partial_u^2 \rho$. Under the more restrictive hypothesis $C^{-1} < \xi_x < C$ (that is, ellipticity of the corresponding random walk), we prove that the non-equilibrium fluctuations from the hydrodynamic limit are given by a non-stationary generalized Ornstein-Uhlenbeck process. As an application, we obtain a central limit theorem for a tagged particle for this model.

Antal Járai (Carleton University):

A self-organized critical forest-fire model

We study a version of a "forest-fire" model introduced by Drossel and Schwabl that in a certain limiting case is expected to show critical behaviour characterized by power laws. We analyze the one-dimensional case whose behaviour is already very interesting, and obtain bounds on the cluster-size distribution. The proofs are rather elementary, but involve subtle geometric constructions.

Elena Kosygina (Baruch College CUNY):

Stochastic homogenization of Hamilton-Jacobi equation with a vanishing viscous term

(Joint work with F. Rezakhanlou and S.R.S. Varadhan)

We consider a homogenization problem for a stochastic Hamilton-Jacobi equation with a vanishing viscous term. We assume that the Hamiltonian is superlinear and convex with respect to the gradient and stationary and ergodic with respect to the spatial variables. Under some additional assumptions on the Hamiltonian we present a homogenization method, which does not use the subadditive ergodic theorem and does not require an a priori uniform gradient estimate.

Michail Loulakis (University of Crete):

Symmetry of the Diffusion Coefficient in Asymmetric Simple Exclusion

We prove the symmetry of the diffusion coefficient that appears in the fluctuation-dissipation theorem for asymmetric simple exclusion processes.

Fabio Martinelli (Università degli Studi Roma 3):

Phase ordering after a deep quench: the stochastic Ising and hard core gas models on a regular tree

(Joint work with P. Caputo)

Consider a low temperature stochastic Ising model in the phase coexistence regime with Markov semigroup P_t . A fundamental and still largely open problem is the understanding of the long time behavior of $\delta_\eta P_t$ when the initial configuration η is sampled from a highly disordered state ν (e.g. a product Bernoulli measure or a high temperature Gibbs measure). Exploiting recent progresses in the analysis of the mixing time of Monte Carlo Markov chains for discrete spin models on a regular b-ary tree $\mathbb{T}^b$, we study the above problem for the Ising and hard core gas (independent sets) models on $\mathbb{T}^b$. If the ν is a biased product Bernoulli law then, under various assumptions on the bias and on

the thermodynamic parameters, we prove ν-almost sure weak convergence of $\delta_\eta P_t$ to an extremal Gibbs measure (pure phase) and show that the limit is approached at least as fast as a stretched exponential of the time t. In the context of randomized algorithms and if one considers the Glauber dynamics on a large, finite tree, our results prove fast local relaxation to equilibrium on time scales much smaller than the true mixing time, provided that the starting point of the chain is not taken as the worst one but it is rather sampled from a suitable distribution.

Thomas Mountford (EPFL):

Exclusion processes with infinite range starting from the heavyside configuration

We know that for an exclusion process whose kernel has finite range and strictly positive mean there exists a so called blocking measure: an equilibrium on configurations with finitely many 1's to the left of the origin and finitely many zeros to the right. We also know that if $\sum_{x<0} x^2 p(x) = \infty$, then there cannot exist such a measure. The question remains open in this case as to whether there exist equilibria with limiting density 1 to the right and density 0 to the left. We discuss a particular, "artificial" case where starting from the heavyside configuration the system converges in distribution to the symmetric convex combination of all 1's and all 0's.

Katalin Nagy (TU Budapest):

On Uniqueness of the Euler Limit of One-Component Lattice Gas Models

(Joint work with József Fritz)

We investigate the interaction of one-dimensional asymmetric exclusion processes of opposite speeds, the exchange mechanism is combined with a spin-flip dynamics, and this asymmetric law is regularized by a nearest neighbour stirring of large intensity. At an intuitive level we can say that particles with ± 1 spins are subject to an external magnetic field. This model admits a single conservation law with hyperbolic scaling. By means of a two-step version of LSI based estimation techniques we prove that compensated compactness and the Lax entropy inequality imply the existence and uniqueness of the hydrodynamic limit. The main purpose of this talk is to demonstrate that attractiveness of the system is not necessary for uniqueness of the hyperbolic scaling limit.

Stefano Olla (Université Paris Dauphine):

On the conductivity and Fourier's law for systems of oscillators with conservative noise

(Joint work with Giada Basile and Cedric Bernardin)

We study the macroscopic conductivity of a system of interacting harmonic oscillators perturbed by a conservative noise. We consider the cases when the noise conserve energy and momentum or only energy.

Hirofumi Osada (Kyushu University):

Interacting Brownian particles related to random matrices

Interacting Brownian particles (IBPs) are (infinitely dimensional) diffusions with free potential Φ and interaction potential Ψ. At least at the intuitive level the diffusions are given by the SDE

$$dX_t^i = dB_t^i - \frac{1}{2}\nabla\Phi(X_t^i)dt - \frac{1}{2}\sum_{j\in\mathbb{Z},\, j\neq i} \nabla\Psi(X_t^i - X_t^j)dt \qquad (i \in \mathbb{Z}) \tag{1}$$

The associated unlabeled dyamics is $\mathbb{X}_t = \sum_{i\in\mathbb{Z}} \delta_{X_t^i} \in \Theta = \{\theta = \sum_i \delta_{x_i}\,;\, \theta(\{|x| \le r\}) < \infty \text{ for all } r\}$. We construct IBPs when their equilibrium measures are Fermion measures associated with sine or Bessel kernels. These measures are coming from the distribution of the spectrum of random matrices.

The difficulty of the problem is the interaction potentials for Fermion measures related to random matrix are logarithmic functions. So we never expect the DLR equation for these measures. To solve the problem we now introduce a notion of *quasi Gibbs measure*:

Definition 1. We say a measure μ on Θ a (Φ, Ψ)-quasi Gibbs measure if it satisfiesF for μ-a.s. η and all $r, k \in \mathbb{N}$ there exist positive constants $c_1 = c_1(\eta, r, k), c_2 = c_2(\eta, r, k)$ such that

$$c_1 \exp\{-\mathcal{H}_r\} d\Lambda_r^k \le \mu_{r,\eta}^k \le c_2 \exp\{-\mathcal{H}_r\} d\Lambda_r^k \tag{2}$$

Here $Q_r = \{|x| \le r\}$, $\pi_r(\theta) = \theta(\cdot \cap Q_r)$, $\pi_r^*(\theta) = \theta(\cdot \cap Q_r^c)$, $\mu_{r,\eta}^k = \mu(\pi_r \in \cdot | \pi_r^*(\theta) = \pi_r^*(\eta), \theta(Q_r) = k)$, Λ_r^k is a Poisson random measure on $\Theta_r^k = \{\theta\,;\, \theta(Q_r) = k\}$, and

$$\mathcal{H}_r(\theta) = \sum_{x_i \in Q_r} \Phi(x_i) + \sum_{x_i \neq x_j \in Q_r} \Psi(x_i - x_j).$$

Theorem 1. *Let μ be a (Φ, Ψ)-quasi Gibbs measure with upper semicontinuous potentials. Then the associated IBPs exist.*

Theorem 2. *Fermion measures with Bessel kernel or sine kernel are quasi Gibbs measures with continuous potentials.*

Oliver Penrose (Heriot-Watt University Edinburgh):

The role of scaling in the microscopic derivation of kinetic equations for interacting bosons

In classical mechanics it was shown by Lanford that one can derive Boltzmann's kinetic equation for a system of hard spheres of diameter a by taking the Boltzmann-Grad limit, in which $a \to 0$ at fixed na^2, where n is the number density. Equivalently, one could take the limit $a \to 0$ at fixed n but re-scale the time and space variables by a factor a^2. The latter version corresponds in quantum mechanics to the scaling limit introduced by van Hove; several authors have used this type of scaling limit in derivations of Boltzmann-type kinetic equations for a Bose or Fermi gas.
I shall argue, however, that these kinetic equations do not help much with the interesting low-temperature behaviour of a Bose hard-sphere system including its possible superfluidity. A better prospect is to use a limit in which $a \to 0$ at fixed na rather than fixed na^2. The resulting kinetic equations (which are completely different) look much more promising from this point of view.

Jeremy Quastel (University of Toronto):

Travelling fronts in some reaction-diffusion processes

Abstract not available.

Fraydoun Rezakhanlou (UC Berkeley):

Coagulation, gelation and Smoluchowski's equation

Consider a model of coagulating mass-bearing Brownian particles. The range of interaction is $N^{1/(2-d)}$ (respectively e^{-N}) if the dimension d is 3 or more (respectively 2), where N is the initial number of particles. The relationship between the mass m and radius r of each particle is given by $r = m^a$. The macroscopic particle densities solve the Smoluchowski's equation provided that $a(d-2) < 1$. We expect to have gelation when $a(d-2) > 1$.

Gunter Schütz (Forschungszentrum Jülich):

Current fluctuations in the zero-range process with open boundaries

(Joint work with R.J. Harris and A. Rákos)

We discuss the asymptotic integrated current distribution of the one-dimensional zero-range process with open boundaries. We observe that the current fluctuations become site-dependent above some critical current and argue that this is a precursor of the condensation transition which occurs in such models. Our considerations for the totally asymmetric zero-range process are complemented by a Bethe ansatz treatment for the equivalent exclusion process (`cond-mat/0504711`, `cond-mat/0506525`).

Timo Seppäläinen (University of Wisconsin):

Fluctuations around characteristics for some many-particle systems

(Joint work with Márton Balázs and Firas Rassoul-Agha)

Some many-particle systems in one dimension have fluctuations on the scale $n^{1/4}$ when the evolution is observed around a characteristic curve of the macroscopic equation. Examples include the random average process and independent walks. In these cases the characteristic curves are parallel straight lines. In a space-time scaling limit the fluctuations converge to a family of Gaussian processes. If the many-particle system is in equilibrium, the time marginal of the limit process is fractional Brownian motion with Hurst parameter 1/4. There are related results for quenched mean processes of certain random walks in random environment. The $n^{1/4}$ scaling picture contrasts with the $n^{1/3}$ scaling with Tracy-Widom limits known for asymmetric exclusion and Hammersley processes.

Sunder Sethuraman (Iowa State University):

Variance estimates for occupation time and a tagged particle in 2D asymmetric simple exclusion

With respect to 2D asymmetric simple exclusion in equilibrium, we consider two problems related in some sense by a common technique. Namely, we give variance bounds on the occupation time at the origin, and for a tagged particle. In particular, we show the occupation time variance is superdiffusive at density 1/2; previously, it has been known that the variance is diffusive when density is not 1/2. However, for a tagged particle, we show the variance is diffusive no matter the density.

Vladas Sidoravicius (IMPA):

Pinning transition in presence of bulk disorder and applications to interacting particle systems

(Based on joint works with V. Beffara, H. Spohn, M.E. Vares.)

I will present few recent ideas and new results related to pinning transition for the directed polymer type models in presence of bulk disorder, and their application to establish critical values of slow-bond problem for TASEP and effects of columnar defect in some (deposition/zero temperature) growth systems.

Herbert Spohn (TU München):

Scaling limit of the stationary TASEP two-point function

(Joint work with Patrik Ferrari)

We consider the space-time covariance of the stationary totally asymmetric simple exclusion process. It is proved that, when moving along the characteristic, the two-point function broadens as $t^{2/3}$. The scaling function is determined and shown to be in agreement with the corresponding scaling function of the stationary PNG model. Thus universality is established, at least in a restricted way. The proof uses determinantal processes and has no overlap with the proof for the PNG model.
The paper is posted at `http://arxiv.org/abs/math-ph/0504041`

Glauco Valle (EPFL):

Convergence results and sharp estimates for the voter model interfaces

We study the evolution of the interface for the one-dimensional voter model. We show that if the random walk kernel associated with the voter model has finite γth moment for some $\gamma > 3$, then the evolution of the interface boundaries converge weakly to a Brownian motion under diffusive scaling. This extends recent work of Newman, Ravishankar and Sun. Our result is optimal in the sense that finite γth moment is necessary for this convergence for all $\gamma \in (0, 3)$. We also obtain relatively sharp estimates for the tail distribution of the size of the equilibrium interface, extending earlier results of Cox and Durrett, and Belhaouari and Mountford.

Srinivasa Varadhan (Courant Institute of NYU):

Homogenization of random Hamilton- Jacobi-Bellman equations

We consider equations of the form

$$u_t = \epsilon \Delta u + H(\nabla u, \frac{x}{\epsilon}, \frac{t}{\epsilon}, \omega)$$

with $u(0,x) = u_0(x)$ We are interested in proving the existence and identification of the limit as $\epsilon \to 0$ of $u = u_\epsilon$. $H = H(p, x, t, \omega)$ is assumed to be convex in p, and a stationary process in t, x. The case when there is no explicit t dependence in H is of interest as well.

Maria Eulalia Vares (CBPF Rio de Janeiro):

On the localization of phases for the one-dimensional random field Kac's model

(Joint work with Marzio Cassandro, Enza Orlandi and Pierre Picco)

The work describes the typical profiles of a one dimensional random field Kac model, in the region of parameters (temperature and magnitude of the field) of two absolute minima for the free energy of the corresponding Curie-Weiss model. We show that, for a set of realizations of the random field of overwhelming probability, the localization of the two phases corresponding to the previous minima is completely determined, and we characterize the transition from one phase to the other.

Bálint Virág (University of Toronto):

Noise limits for Coulomb gas, zeros and eigenvalues

(Joint work with Brian Rider)

Consider a Coulomb gas in the plane: n points distributed with density proportional to product $|x_i - x_j|^2$ with respect to some underlying measure μ.
When μ is Gaussian, this process coincides with the eigenvalues of matrix filled with i.i.d complex normal entries. Other choices of μ yield the zeros of i.i.d. Gaussian power series, the projection of the spherical gas, or the eigenvalues of a random unitary matrix. These processes converge to noises that are different form the usual white noise. Reporting on joint work with Brian Rider, I will expain which, how and why.