Aspects of Counterparty Credit Risk Valuation

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Agenda

1. What is credit risk?
2. How to mitigate it?
3. What is CVA?
4. How to compute it?
5. A few example products and how to compute CVA for these
6. A practical challenge

What is credit risk?

- Financial contract ⟶ future cashflows

  E.g.:
  - Corporate bond
  - European Option
- Traditional pricing theory: we take future cashflows granted
- What if my counterparty defaults?
  - If I owe them I just pay it back at market price
  - If they owe me I might get back some of my money

How to mitigate it?

- Netting
- Collateralization
  - Repurchase agreement (Repo)
- Clearing
  - Standardized contracts, e.g. Interest Rate Swaps
  - Variational margin, can be e.g. daily, weekly
  - Initial margin
- Hedging with Credit Default Swaps (CDS)
  - In case of a credit event, the issuer pays an agreed amount
  - In return, the buyer pays a regular premium

What is CVA?

- Credit Valuation Adjustment
- Spread on contract price to cover credit risk due to:
  - Lack of other type of credit risk mitigation
  - Or residual risks, e.g.
    - Clearing happens only time-to time
    - Margin call happens only if exposure exceeds a limit

How to compute CVA?

3 main components:

- **Default distribution**: probability of default around a future date
  - We assume an inhomogeneous Poisson process
  - Default rates are calibrated to CDS prices
- **Exposure distribution**: my exposure around a future date
  - Expected *Positive* Exposure (EPE)
  - E.g. Monte Carlo simulation
- **Recovery rate**
  - For simplicity say its constant, 40%

How to compute CVA? – unilateral formula [2]

$$CVA(t) = E(\,(1-R)(V(\tau))^+ \mid F_t\,) =$$

discounting

$$= (1-R)\int_{s=t}^{T} \lambda(s)e^{-\int_{u=0}^{s}\lambda(u)du}\, E\left(e^{-\int_{u=0}^{s} r(u)du}(V(s))^+ \,\Big|\, G_t\right) ds$$

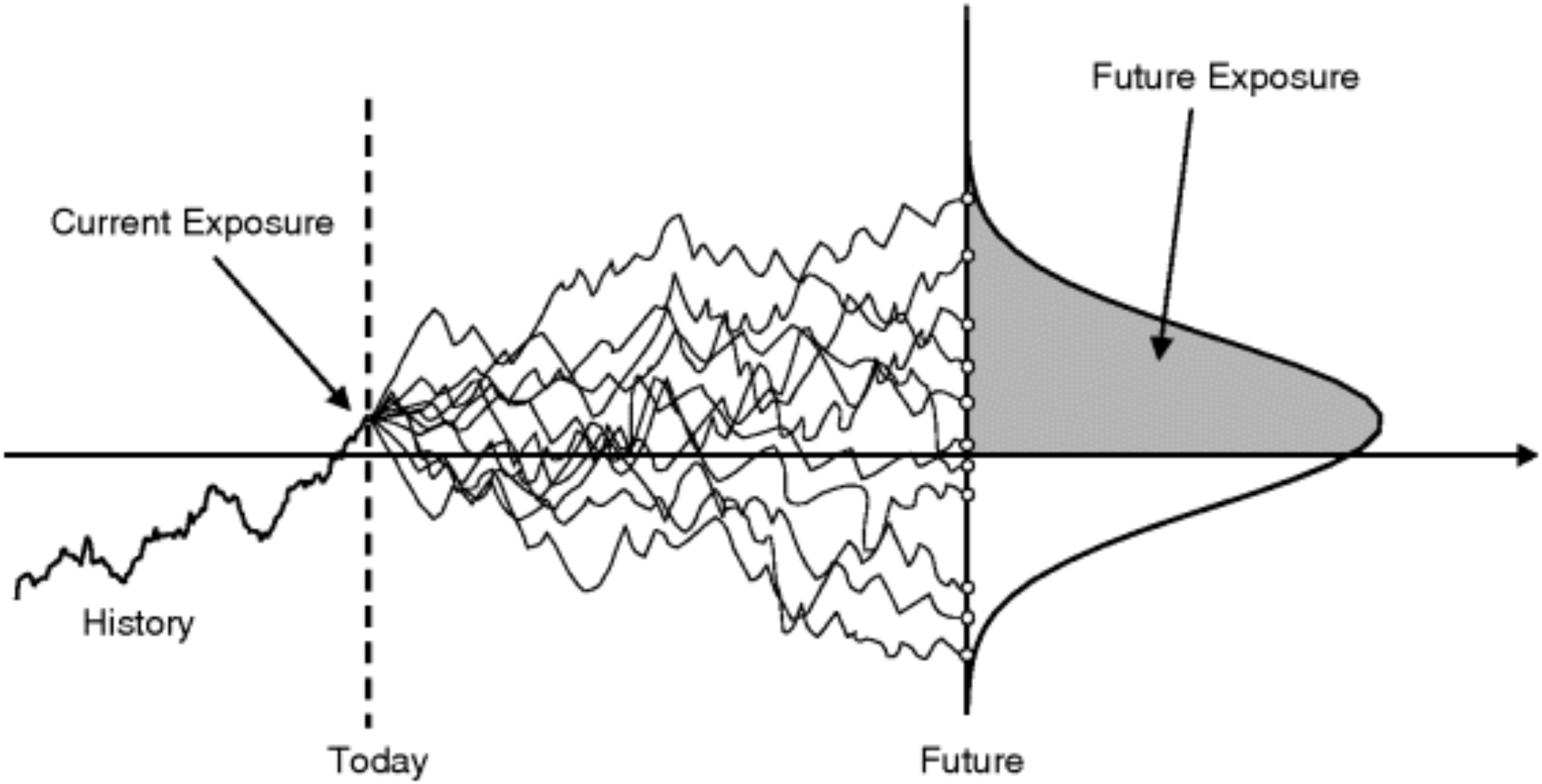
default (Poi process)

Exposure (EPE)

Where:

- $\boldsymbol{\tau}$ is the (random) time of default, $\boldsymbol{T}$ is the maturity of the contract
- $\boldsymbol{R}$ is the recovery rate, $\boldsymbol{V(s)}$ is the (random) price of the contract
- $\boldsymbol{F_t}$ is the filtration containing all the info up to time t
- $\boldsymbol{\lambda(s)}$ is the hazard rate
- $\boldsymbol{r(u)}$ is the discount rate
- $\boldsymbol{G_t}$ is the filtration containing info needed for exposure calculation

What is EPE?



Example: European Option

European call: We have an *option* to buy q shares of SP500 for $\$K$ each at a forward time T.

Payoff at maturity:

$$q(S(T) - K)^+$$

Can be priced using an analytic model

(Black-Scholes, geometric Brownian motion)

Price at maturity and before: lets draw…

Example: CVA for European Option

How to compute CVA?

- Discretize the integral
- For exposure:
  - Simulate the underlyings (e.g. SP500) with a MC model
  - Underlying simulation can be correlated with the default process (Wrong way risk)
  - On each path for each CVA date the contract can be priced using B-S formula
- Multiply exposures with prob. of default in given time interval

Example: Range Accrual

- A premium is paid for days when underlying falls in an interval (L, H)
- Payoff at maturity:

$$\frac{C}{n} \sum_{t \in \{t_1, \dots, t_n\}} 1_{\{L < S(t) < H\}}$$

- Path dependent
- Pricing with Monte Carlo
- Draw…

CVA for Range Accrual – a practical challange

Try the same as before:

- Discretize the integral
- For exposure:
  - Simulate the underlyings with a MC model
  - On each path for each CVA date price the contract

$\longrightarrow$ that’s MC in MC, super time-consuming

Longstaff-Schwartz method

Longstaff-Schwartz [1]:

- Instead of a new MC on each MC path we would like to use the continuation of existing paths somehow
- How to use existing paths? Why is it not trivial? Lets draw…

Longstaff-Schwartz method

We do a tricky linear regression:

Target: price of the contract at a CVA date s on a path, i.e.

$$E(V(s) \mid S(s) = x)$$

Regressors: const, underlying value $S(s)$, its functions $f(S(s))$,etc.

Numeric example

European call, $K = 10.1, T = 10$

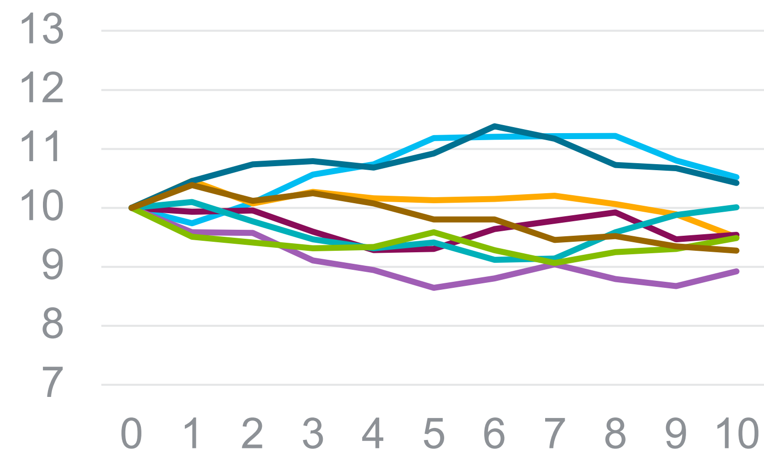
$(S(T) - K)^{+} - 0.3$

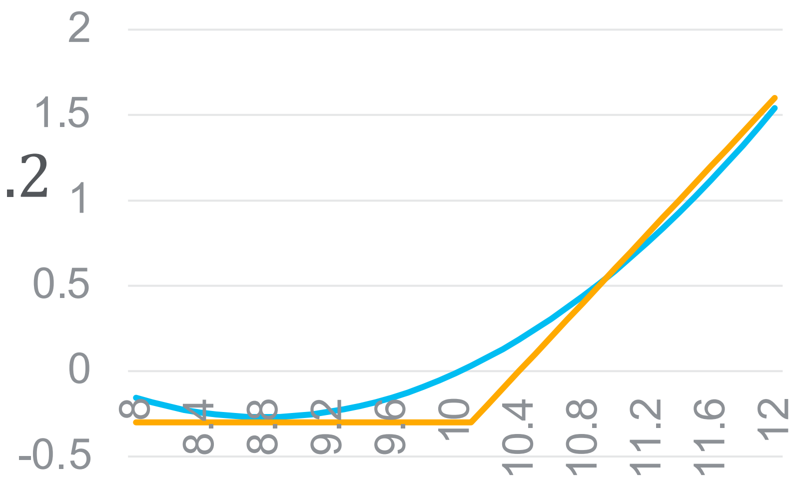
What is EPE at $t = 4$?

Regression: $\mathrm{p(x)} = \mathrm{a} + bx, +cx^2$

where $x = S(4)$ will be used

Calibration: $a = 13.3, b = -3.1, c = 0.2$





Numeric example cont.

- LSM is calibrated
- MC simulation: 8 paths ☺
- Substitute to $p(x)$

  ⟶ exposure estimation

- Floor to zero to get positive exposure
- take average to get EPE

$S(4)$	$p(S(4)$	pos. exp.
10.74	0.40	0.40
10.16	0.06	0.06
9.28	-0.23	0.00
8.94	-0.26	0.00
9.32	-0.22	0.00
9.34	-0.22	0.00
10.68	0.36	0.36
10.08	0.02	0.02

$$EPE = 0.10$$

Literature

- [1] Francis A. Longstaff and Eduardo S. Schwartz
  Valuing American Options by Simulation: A Simple Least-Squares Approach,
  The Review of Financial Studies, 2001

- [2] Andrew Green
  XVA: Credit, Funding and Capital Valuation Adjustments,
  The Wiley Finance Series, 2015

Books


WILEY FINANCE
Counterparty credit risk and credit value adjustment
A continuing challenge for global financial markets
Second Edition
JON GREGORY
WILEY FINANCE
Counterparty credit risk, collateral and funding
With Pricing Cases for All Asset Classes
DAMIANO BRIGO
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Counterparty Risk and Funding
A Tale of Two Puzzles
Netting Set MTM vs Time
Chapman & Hall/CRC FINANCIAL MATHEMATICS SERIES
The xVA Challenge
Counterparty Credit Risk, Funding, Collateral and Capital
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