A Dollar and Another

Thoughts about Mental Accounting

Gábor Salamon

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- Earned as salary
- Gained as unrealized investment profit

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- Paid as tax (deducted from salary / transferred from account)
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Examples from Richard Thaler

- Mr and Mrs A went for a fishing trip. They sent home the salmons they caught by plane. Their package got lost, they were given a $300 compensation. From this amount, they went for a dinner which cost them $225. They never spent such an amount for a restaurant meal before.

- Breaches the substitution principle. $300 was considered as unexpected gain and was already ”mental-booked” to the ”food account”.

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Examples from Richard Thaler

- Mr B and Mr C are playing poker. Mr B is currently in a profit of $50, while Mr C is at even but he has just won a substantial amount on his IBM stocks. Mr B has a queen straight and raises by $10. Mr C has a king straight and folds. He thinks: ”If I had been in a profit of $50, I would have raised, too”.

- Mental accounts are separated, in terms of source funding, goal and timing of spending.

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Examples from Richard Thaler

- Mr and Mrs D have been saving \$15000 so far to buy a weekend cottage. They are planning to buy the house in 5 years. The yield on their money market found is 10%. They have just bought a new car for \$11000 which they financed with a 3-year 15%-interest loan.

- Again, the substitution principle is breached. The couple does not trust in their own self-control, therefore they use the bank to "force" them keep the saving schedule.

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Examples from Richard Thaler

- Mrs E likes a sweater which she found in a shop for $125. She did not buy it as she thought it was very expensive. A few weeks later his husband gives her the same sweater as a birthday gift which makes her very happy. All they bank accounts are shared.

- Gifts received are priced differently than when the same object is bought. Emotional utility is playing a role.

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Examples from Richard Thaler

- Mr F decides in January to donate \$2000 to his preferred charity in December. Whenever a sudden misfortunate event happens to him during the year, he pays the cost from this sum. He donates in December whatever rests from that \$2000.

Overview

- Thoughts about utility
- von Neumann-Morgenstern utility
- Prospect Theory's value function
- A mental accounting model
- Transactional utility
- From an investor's perspective
- Xmas boxing

A Classic Model: Maximizing Utility

- n products
- prices: c_i
- amount consumed: z_i
- income: I
- utility: $U(z)$

$$\max_z U(z) : \sum_i c_i z_i \leq I$$

Types of Utility

- Financial (money)
- Hedonic (emotions, self-image)
- Expressional (social image)

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Bernoulli game

Example

How much would you pay for this game?

- Nicolas Bernoulli 1713
- Saint Petersburg paradox
- expected value: infinite (n after n step)
- solution ?
  - there is a utility function ensuring a finite utility even for an infinite payout
  - small probability events are simply ignored
  - finite version: with Bill Gates – about $22.

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Expected Utility Theory

- Daniel Bernoulli 1738
- Payout
  - Utility of payout (subjective EU: not necessarily financial)
  - Probability of payout
- von Neumann, Morgenstern: necessary and sufficient conditions under which it holds
- risk aversion implies a concave utility function

Utility and Risk attitude

Risk Neutral

$U(0.5) = \frac{1}{2} \cdot U(0) + \frac{1}{2} \cdot U(1)$

Risk Averse

$U(0.5) > \frac{1}{2} \cdot U(0) + \frac{1}{2} \cdot U(1)$

Risk Seeking

$U(0.5) < \frac{1}{2} \cdot U(0) + \frac{1}{2} \cdot U(1)$

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von Neumann–Morgenstern Utility Theorem

Definition

A lottery L is composed of pairs, each with a payout c_i and a probability p_i
Axioms:

- completeness

$$\forall L, M : L \geq M \text{ or } M \geq L$$

- transitivity

$$\forall L, M, N : L \geq M, M \geq N \rightarrow L \geq N$$

- independence

$$\forall L, M, N, t \in [0, 1] : L \geq M \rightarrow tL + (1-t)N \geq tM + (1-t)N$$

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$$\forall L, M, N : L \geq M \geq N \rightarrow \exists p \in [0, 1] : M = pL + (1-p)N$$

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Theorem

If all of these hold, the individual is considered rational and a utility function U can be constructed such that choosing the highest expected utility lottery is equivalent to choosing the lottery which maximizes $\geq$, that is

$$L \geq M \leftrightarrow E(U(L)) \geq E(U(M)).$$

von Neumann–Morgenstern Utility Theorem

Criticism

- cannot explain loss aversion
- same wealth does not imply same happiness
  - how happy would you or Bill Gates be with a total wealth of $10 million?
- conservatism in updating beliefs (hard to deal with probabilities)
- framing dependent utility
- Allais paradox (independence breached)
- Ellsberg paradox (nested gambling: risk perception about risk itself)

Allais paradox

Example

Lotteries

A $1M

B $1M with 89% probability
$5M with 10% probability
$0 with 1% probability

C $1M with 11% probability
$0 with 89% probability

D $5M with 10% probability
$0 with 90% probability

We only substracted ($1M,89%)
Experimental results: $A \geq B$ and $D \geq C$
Breaches the independence axiom of von Neumann-Morgenstern utility.

Allais paradox

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Ellsberg paradox

Example

1 urn, 30 red balls, 60 other balls: either yellow or black (any mix possible)
Lotteries

- A $100 if drawing a red, 0 otherwise
- B $100 if drawing a black, 0 otherwise
- C $100 if drawing a red or yellow, 0 otherwise
- D $100 if drawing a black or yellow, 0 otherwise

Experimental results: $A \geq B$ and $C \leq D$
Explanation: ambiguity aversion (where probabilities are unknown: Knightian uncertainity)
See also: risk versus uncertainity

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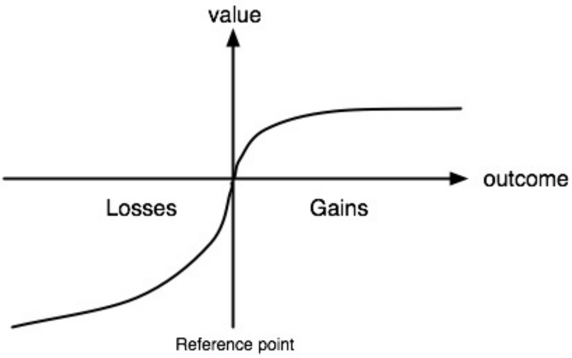
Prospect Theory’s Value Function



- Reference point, losses and gains, no absolute utility
- Convex for losses and concave for gains
- Steeper for losses than for gains

Coupling and Separation Gains and Losses

Two transactions: $x > 0$ and $y > 0$ (a loss: $-x$ and $-y$)
Prospect theory's value function v

1. Multiple gains: $v(x) + v(y) > v(x + y)$.
   Separate
2. Multiple losses: $v(-x - y) > v(-x) + v(-y)$.
   Couple
3. Mixed gain x and loss $-y$: suppose $x > y$, then $v(x) + v(-y) < v(x - y)$.
   Couple
4. Mixed loss $-y$ and gain x: suppose $y > x$, then both coupling and separation can be favorable, depending on x and y.
   If $x << y$, Separate (consolation)

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Transactional Utility Theory

- Product value z
- Actual price c
- Indifference price $\bar{c}$
- Anticipated fair reference price c^*

Purchase utility: $v(z, -c) = v(\bar{c}, -c)$
Transactional utility: $v(-c \mid -c^*)$, that is, c^* is anticipated, c is paid
Value of buying at market price c while we anticipated reference price c^*:

$$w(z, c, c^*) = v(\bar{c}, -c) + v(-c \mid -c^*)$$

Why people were willing to pay \$2.65 for a beer from a fancy beach hotel while they offered only \$1.50 for the same beer bought from a small shop near the beach?

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Mental Account Types

- Current Income
- Current Wealth
- Future Income

Further Examples

- Stock premium puzzle (around 6% historically): portfolio evaluation frequency (myopic loss aversion)
- Opening and closing positions (paper vs realized P&L)
- Payment decoupling: flat price vs pay-as-you-go (credit cards)
- Notice expenses first: small amounts are not booked to mental accounts (27c a day vs $100 a year)

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Investor mistakes

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- Underdiversification due to thinking of separate accounts
- Seeking for income over capital gain (dividend and its tax disadvantages)

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Investor Benefits

Self-Control Tool

Goal Based Planning

- Needs and Obligations $\longrightarrow$ LOW risk
- Priorities and Desires $\longrightarrow$ MEDIUM risk
- Aspirations $\longrightarrow$ HIGH risk

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