CSFK

Time-series analysis in variable star astronomy: recent advances in the physics of stellar oscillations

László Kiss
Research Centre for Astronomy and Earth Sciences

BME Mathematical Modelling Seminars 2019

In this talk

- Introducing stellar oscillations
- Observational work:
    - radial velocities
    - space photometry
- **The linear regime:** Fourier-transforms everywhere
- **The non-linear regime:** evidence of chaos in some oscillating stars

From red giants…

1580 *E. Bányai et al.*

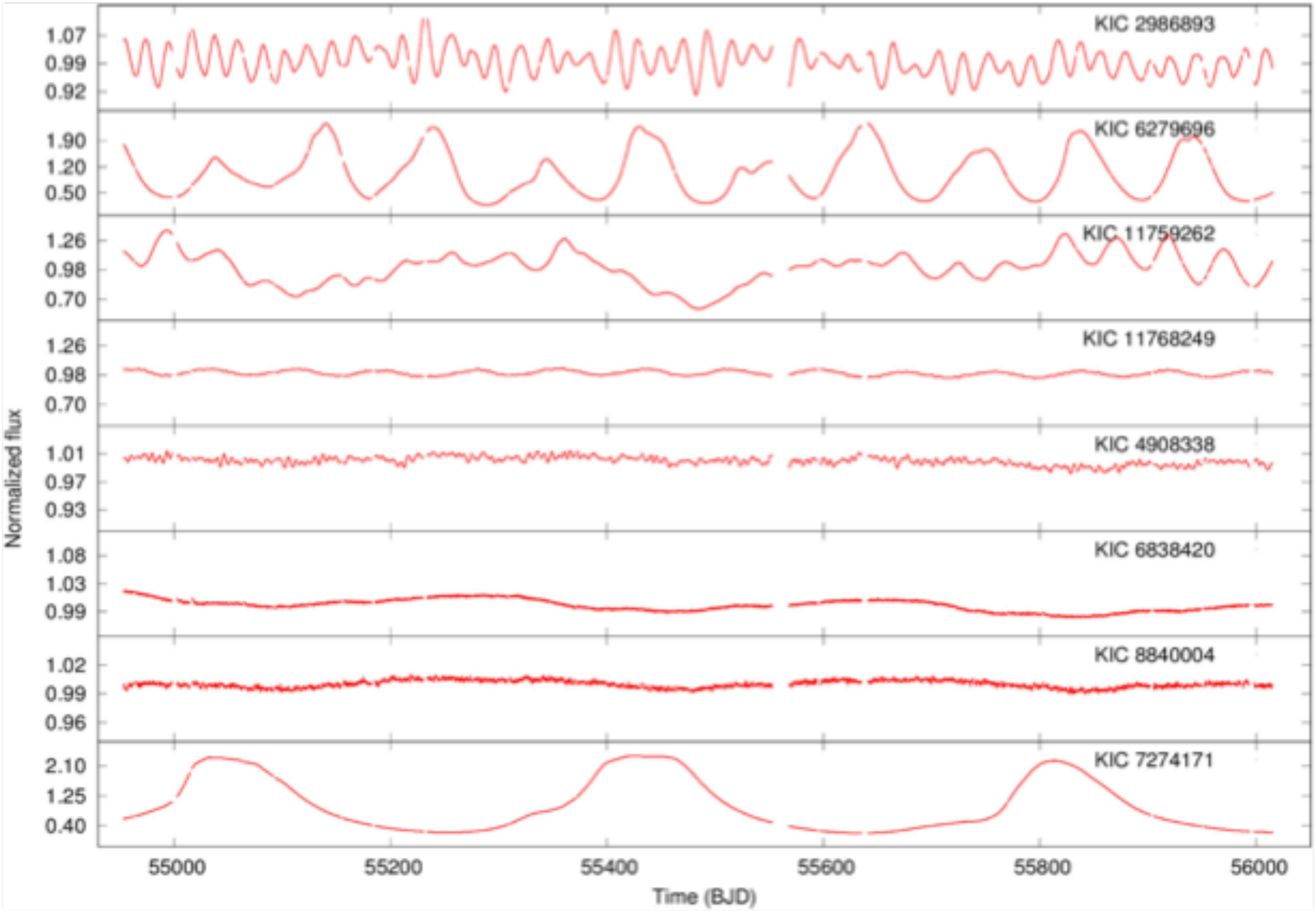
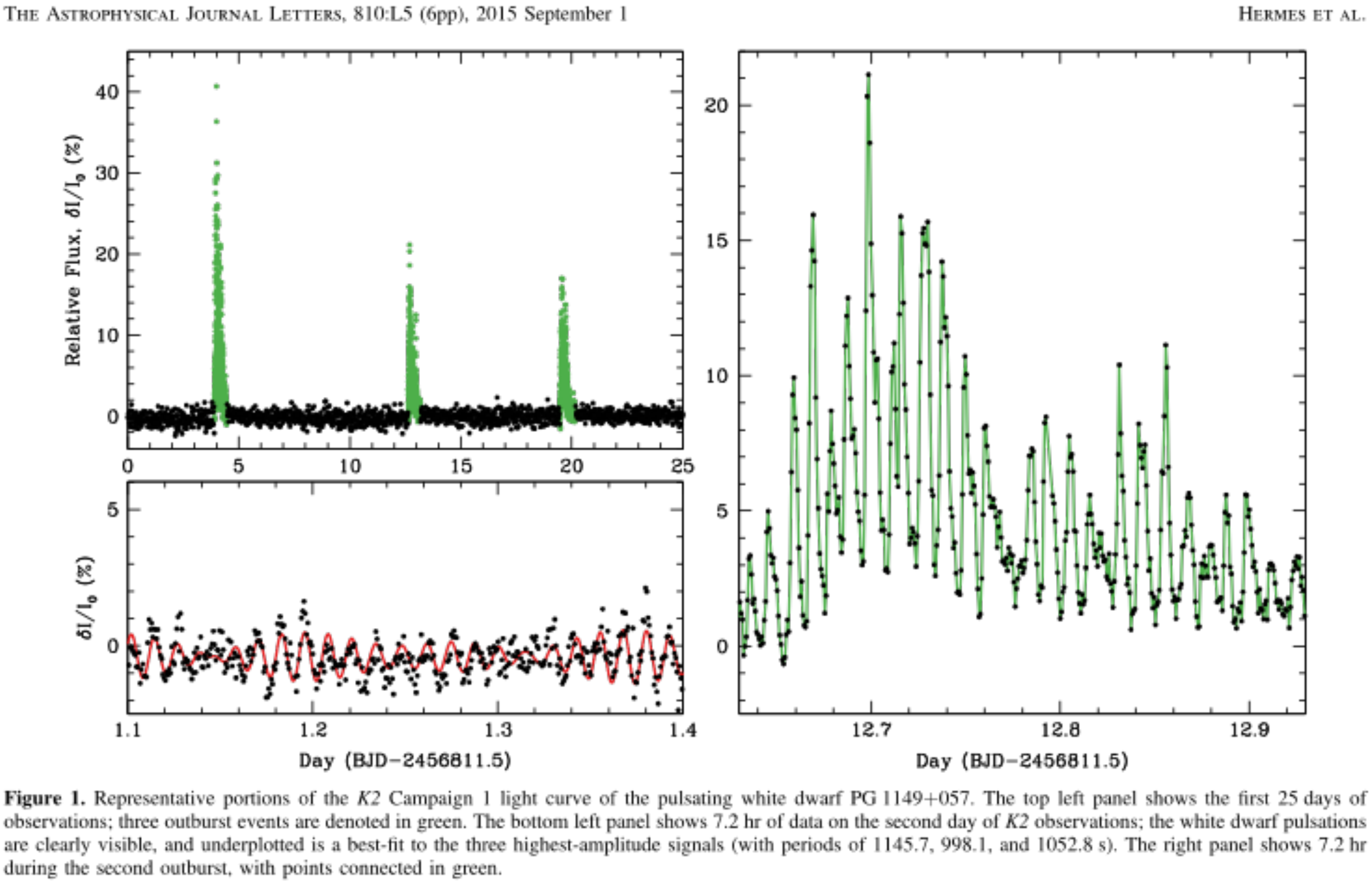


Figure 4. Data for the same stars as in Fig. 2 after the correcting procedure.

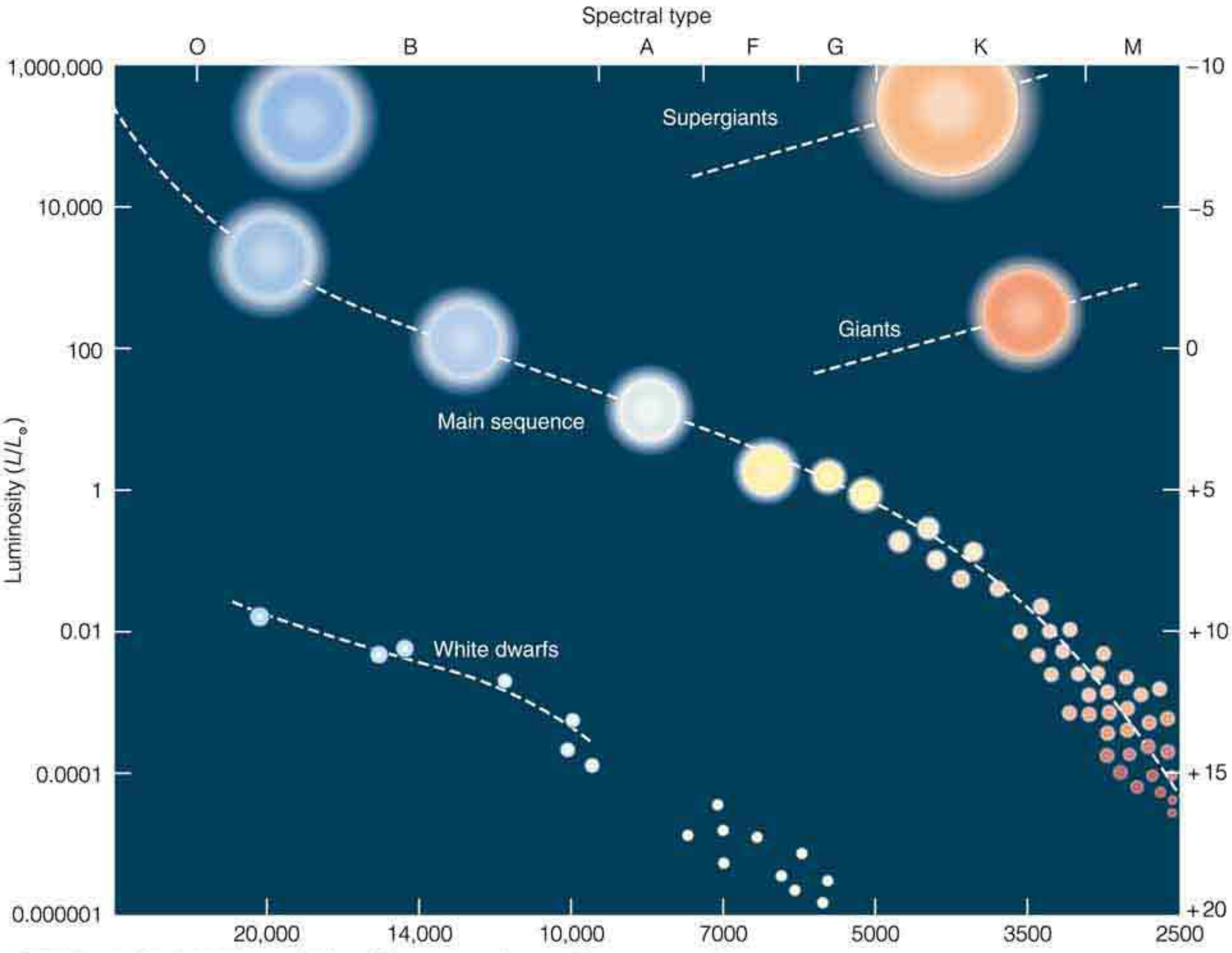
…to white dwarfs



THE ASTROPHYSICAL JOURNAL LETTERS, 810:L5 (6pp), 2015 September 1 HERMES ET AL.

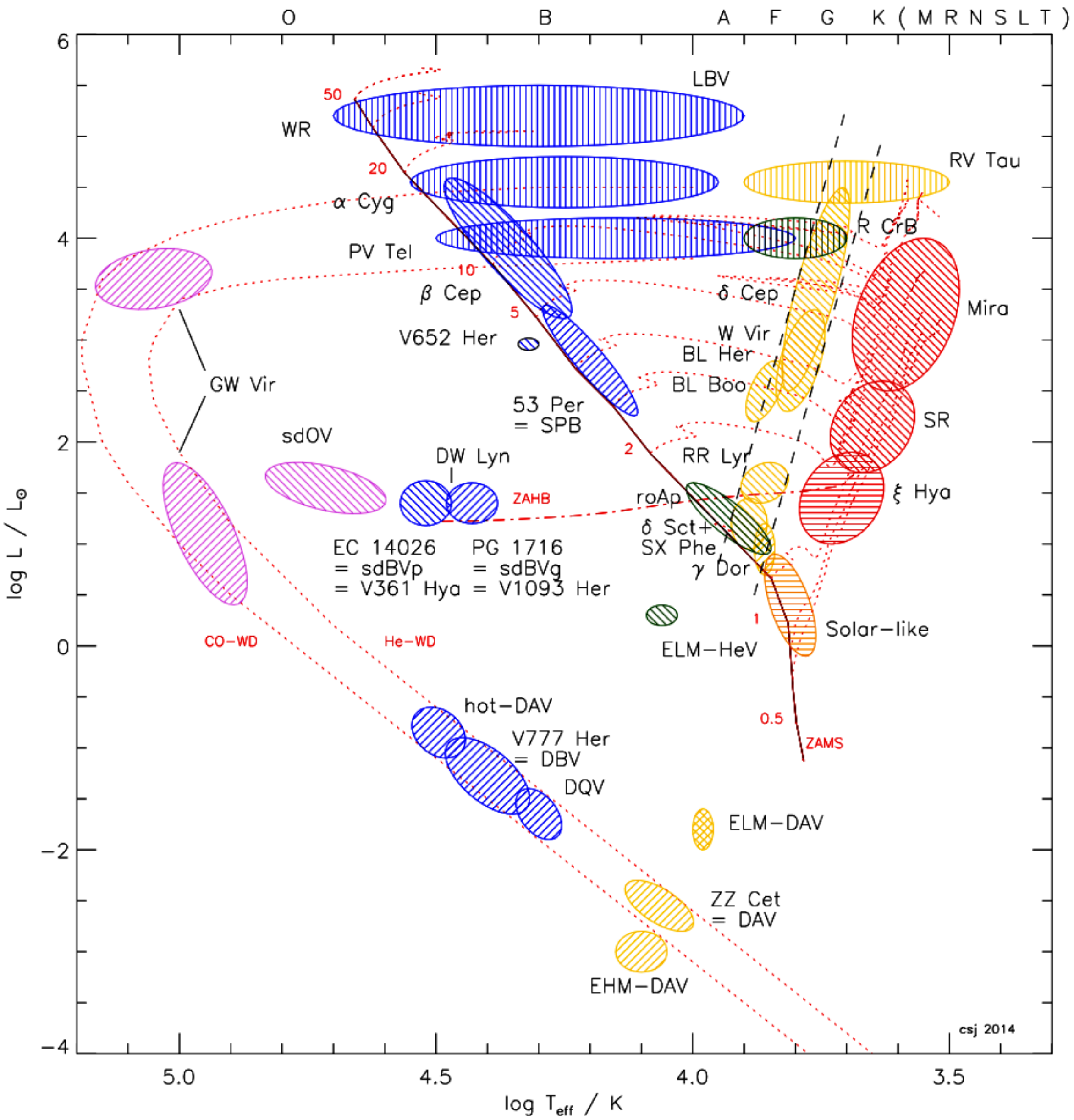
Figure 1. Representative portions of the *K2* Campaign 1 light curve of the pulsating white dwarf PG 1149+057. The top left panel shows the first 25 days of observations; three outburst events are denoted in green. The bottom left panel shows 7.2 hr of data on the second day of *K2* observations; the white dwarf pulsations are clearly visible, and underplotted is a best-fit to the three highest-amplitude signals (with periods of 1145.7, 998.1, and 1052.8 s). The right panel shows 7.2 hr during the second outburst, with points connected in green.

Hertzsprung–Russell Diagram (HRD)



The zoo of pulsating stars

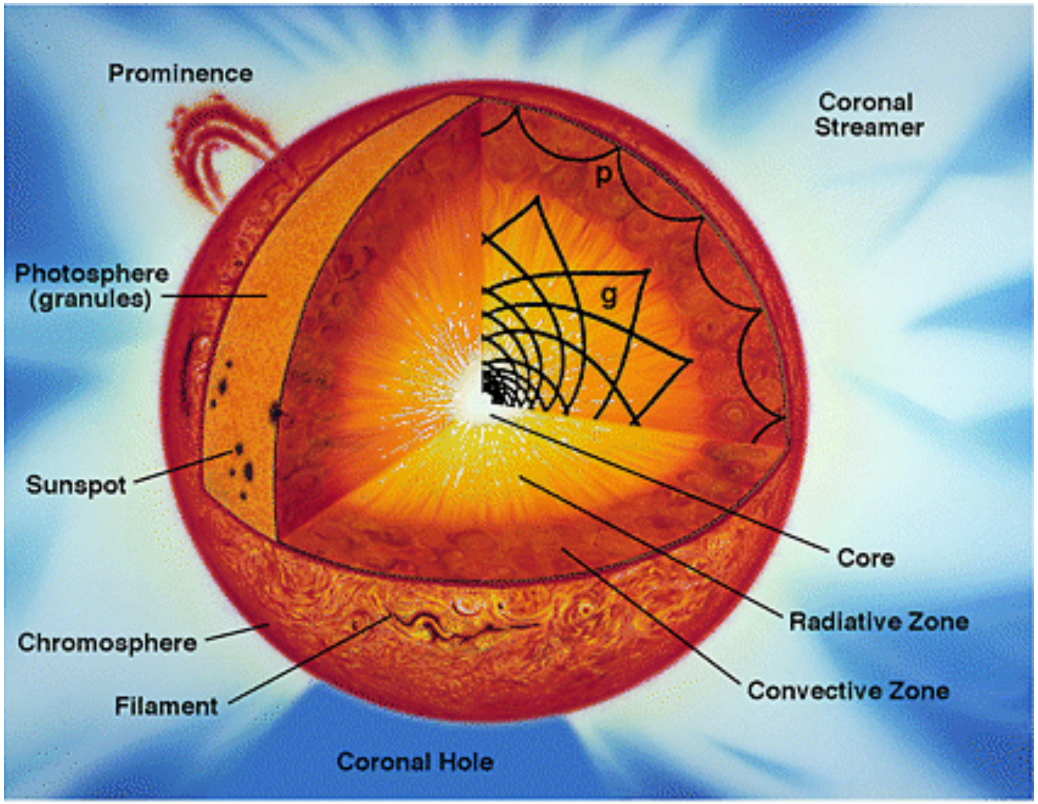
C. S. Jeffery & H. Saio



Stellar structure: stars as centrally condensed objects

- energy producing core (shells)
- radiative zone
- convective zone
- atmosphere interactions:

energy flow + gravity = pulsations



NASA/Goddard Space Flight Center

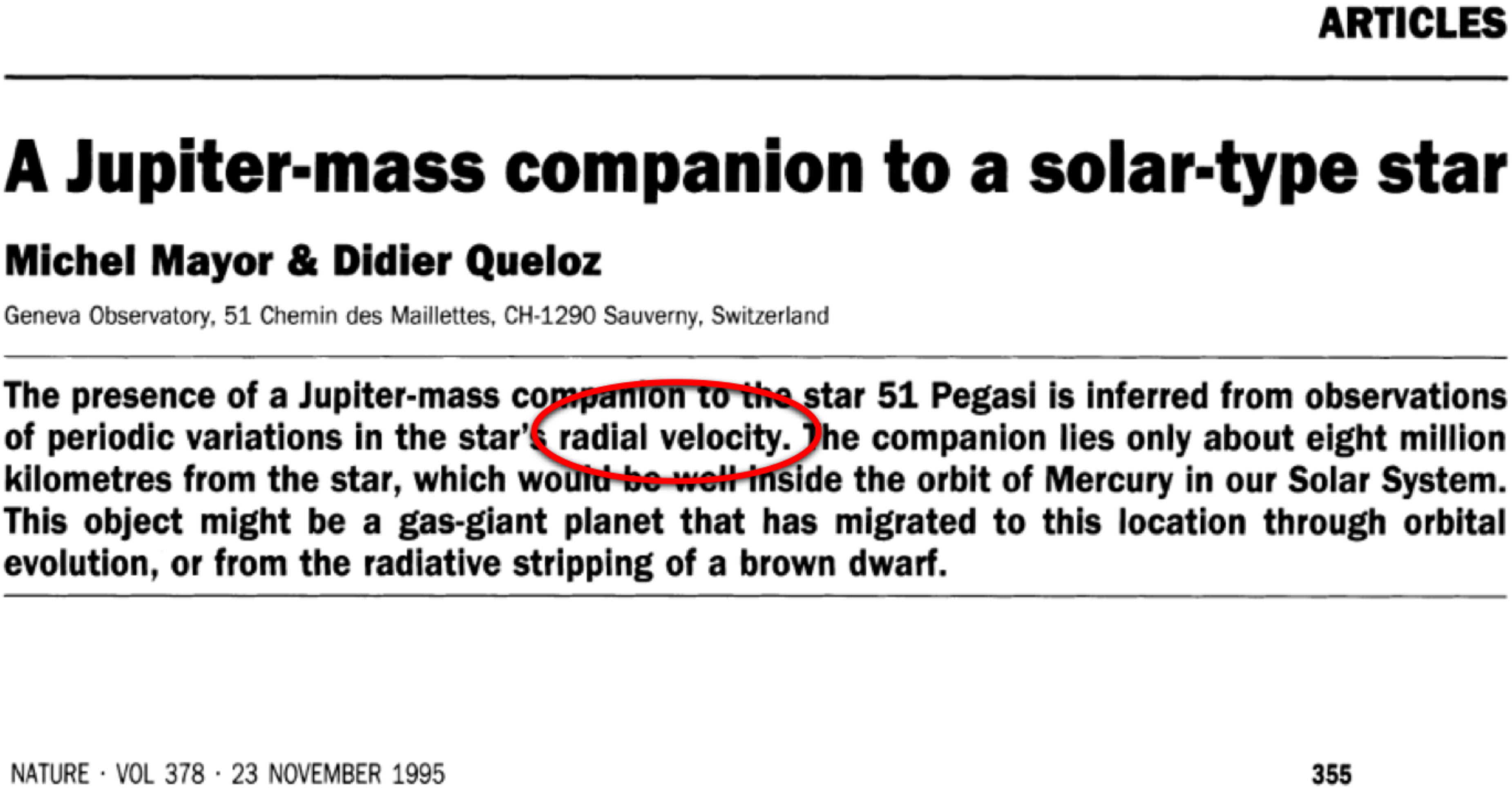
What are stellar oscillations good for?

- understanding the physics of pulsations (like driving and damping mechanisms)
- determining stellar properties with surprising precision and accuracy (mass, radius, age, internal rotation, distance, etc.)
- understanding matter’s behaviour under extreme physical conditions (c.f. plasma opacities, solar neutrino problem)

How can we measure oscillations of stars?

- velocity variations
- brightness variations

Exoplanets: since 1995



ARTICLES

A Jupiter-mass companion to a solar-type star

Michel Mayor & Didier Queloz

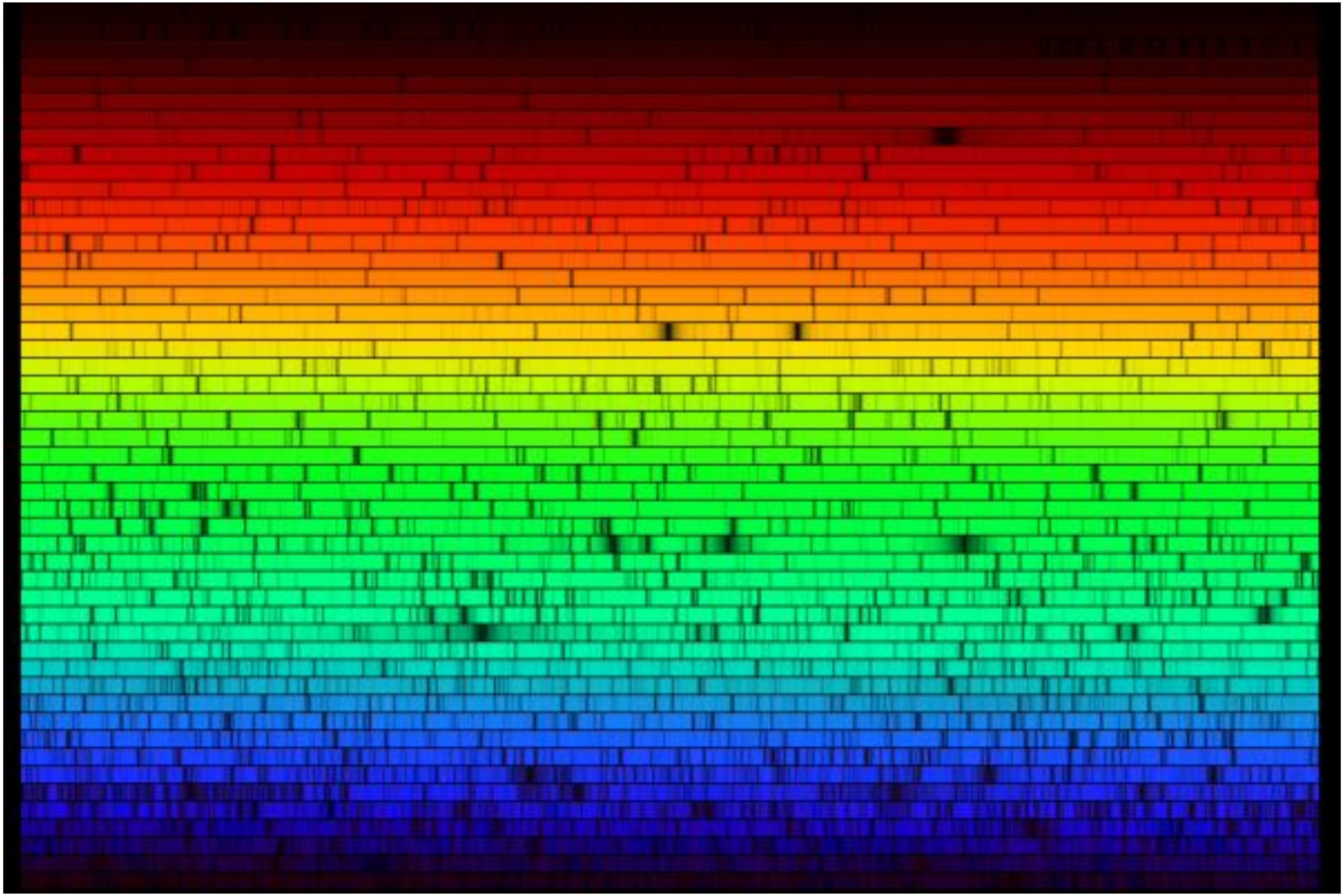
Geneva Observatory, 51 Chemin des Maillettes, CH-1290 Sauverny, Switzerland

The presence of a Jupiter-mass companion to the star 51 Pegasi is inferred from observations of periodic variations in the star's radial velocity. The companion lies only about eight million kilometres from the star, which would be well inside the orbit of Mercury in our Solar System. This object might be a gas-giant planet that has migrated to this location through orbital evolution, or from the radiative stripping of a brown dwarf.

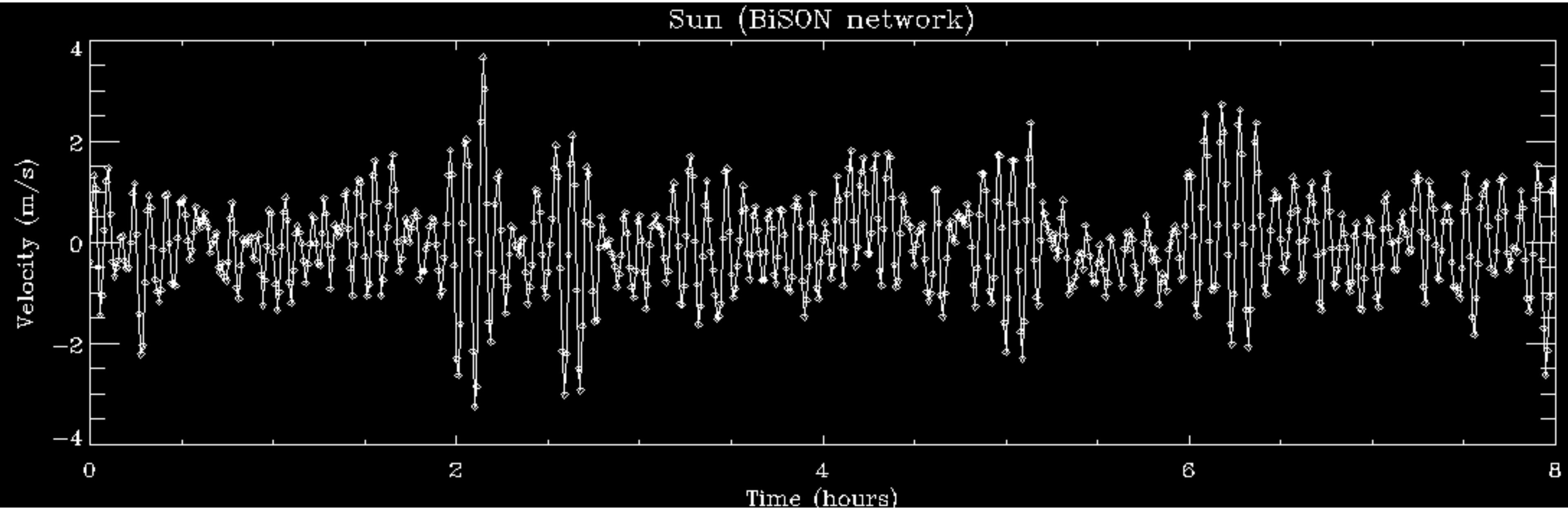
NATURE · VOL 378 · 23 NOVEMBER 1995

355

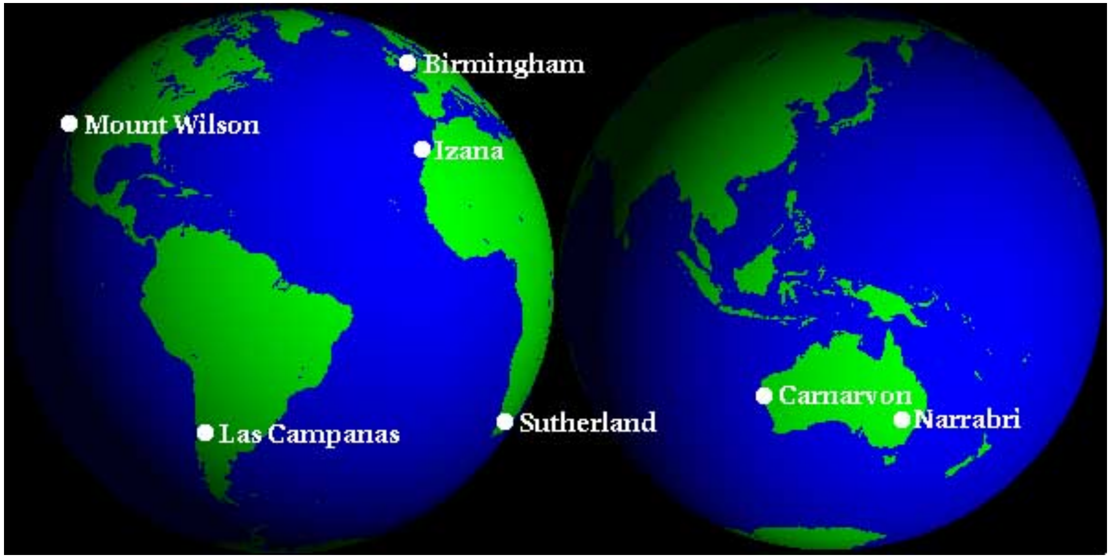
The solar spectrum



Velocity fluctiations of the Sun



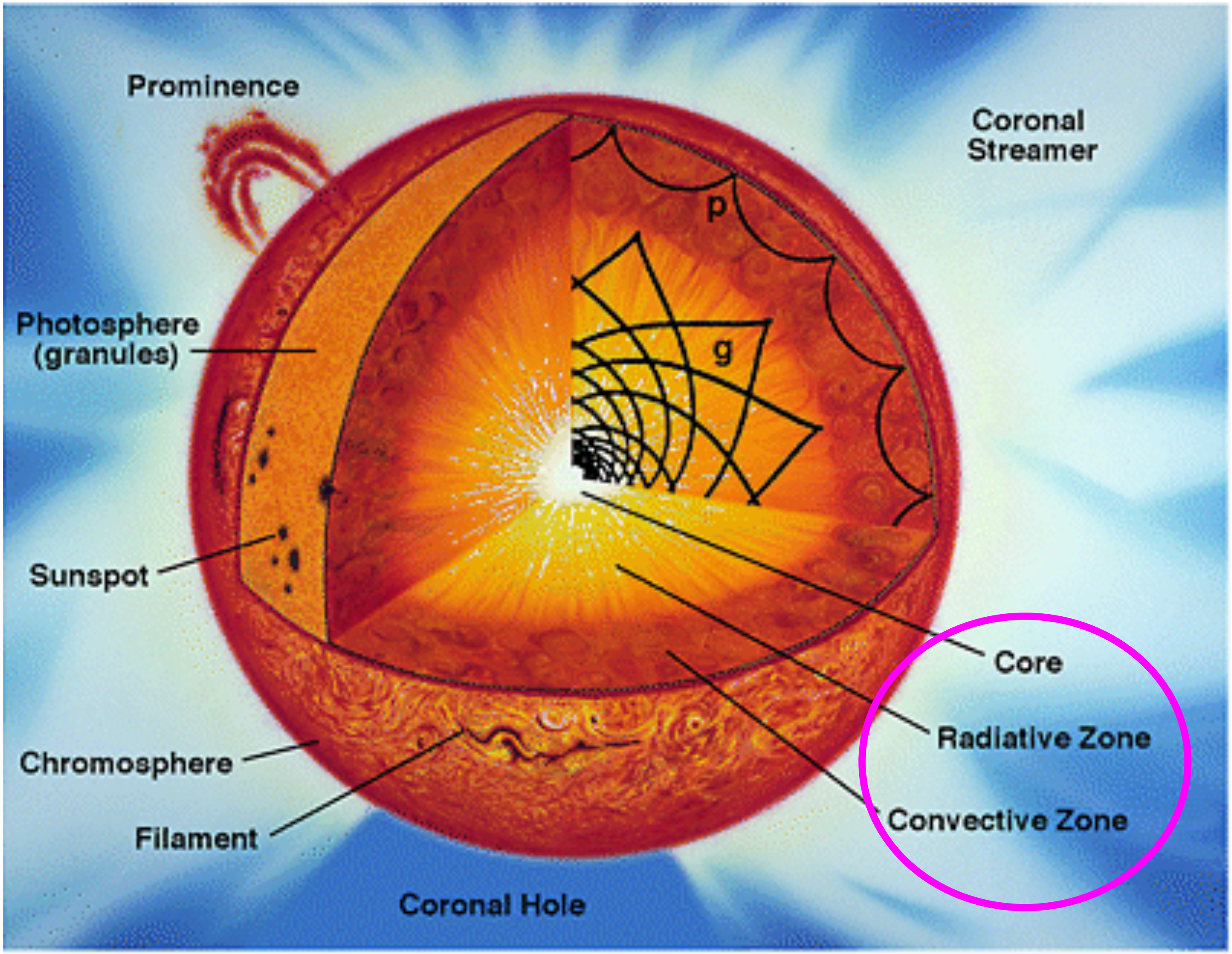
BiSON (Birmingham Solar Oscillations Network)



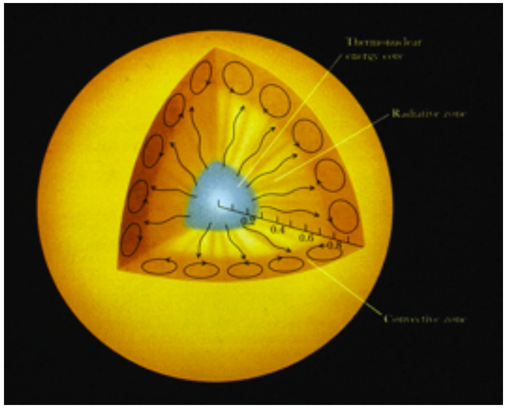
What kind of oscillations?

- sound waves (**p-modes**)
- gravity waves (**g-modes**)
(NOT the gravitational waves in spacetime)

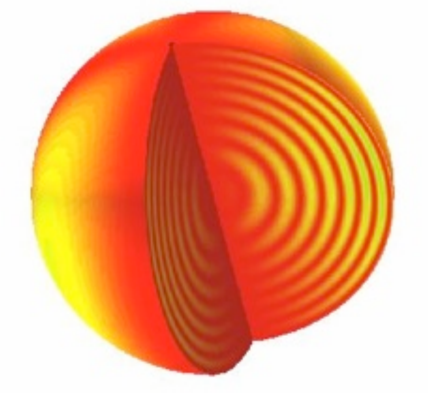
Within the Sun



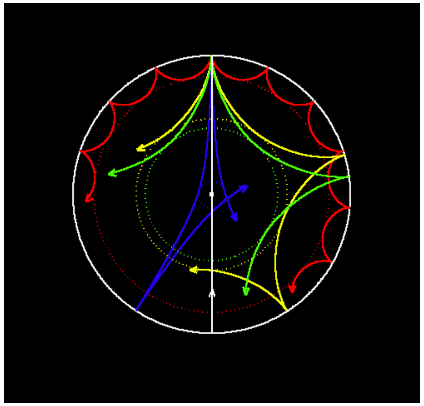
Solar-like oscillations



The convective zone excites oscillations near the surface.



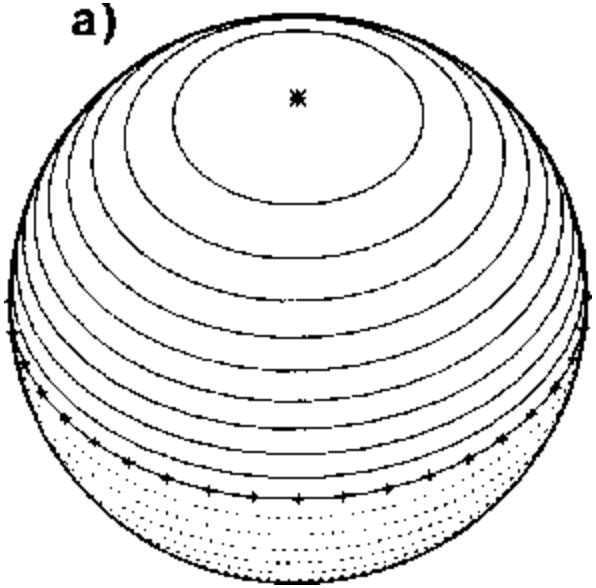
The modes correspond to the eigenmodes of a spherical organ pipe. Radial and non-radial oscillations.

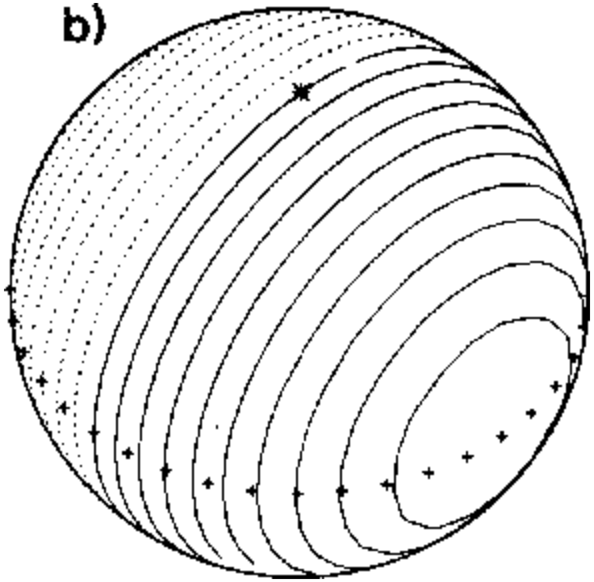


By measuring frequencies we reveal the internal structure because certain waves penetrate deep inside the stars.

$$c^2 \simeq \frac{\gamma \cdot k_B \cdot T}{\mu \cdot m_u}$$

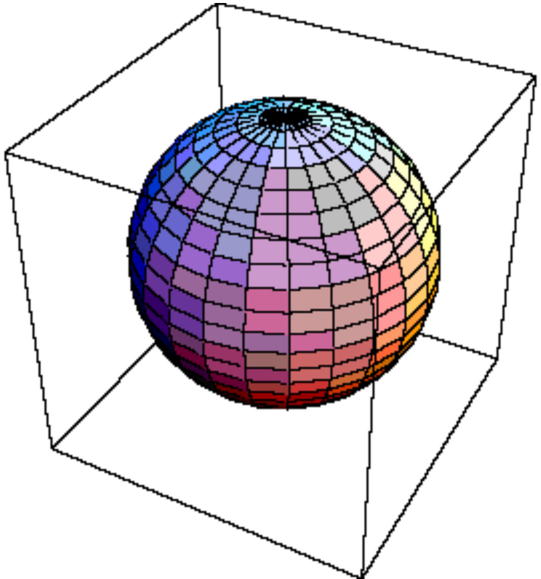
Spherical harmonics

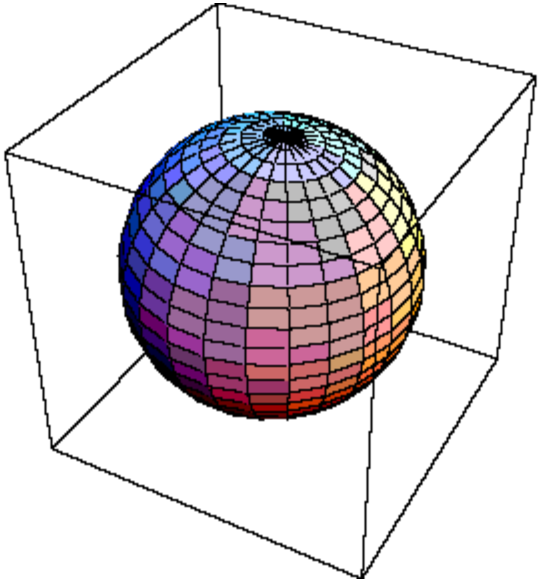


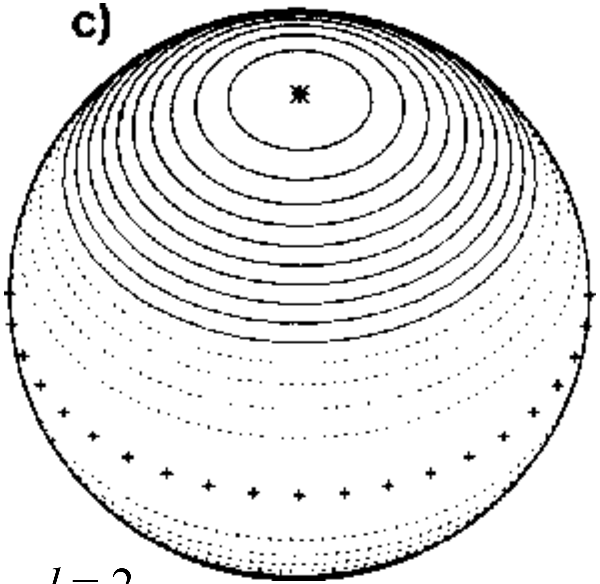


$l = 1$
$m = 0$

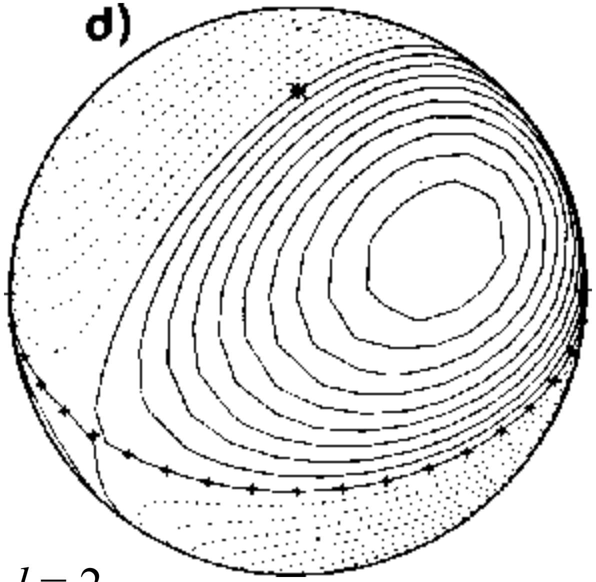
$l = 1$
$m = 1$



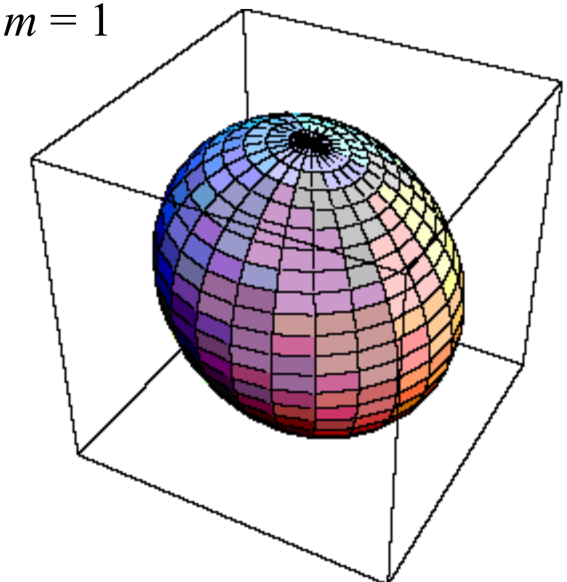


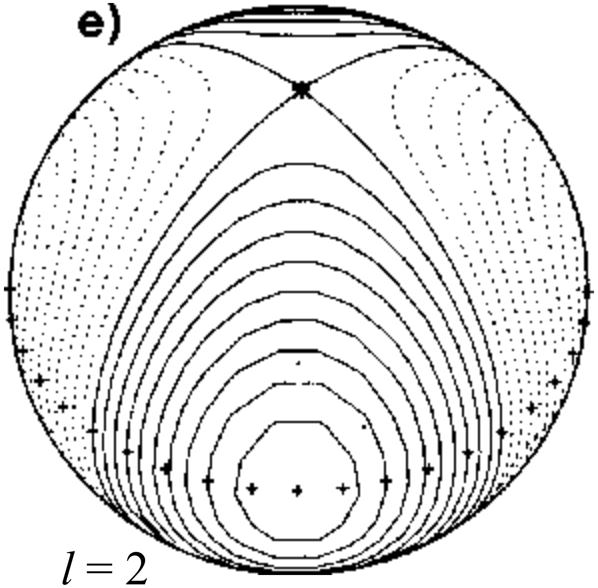


$l = 2$
$m = 0$

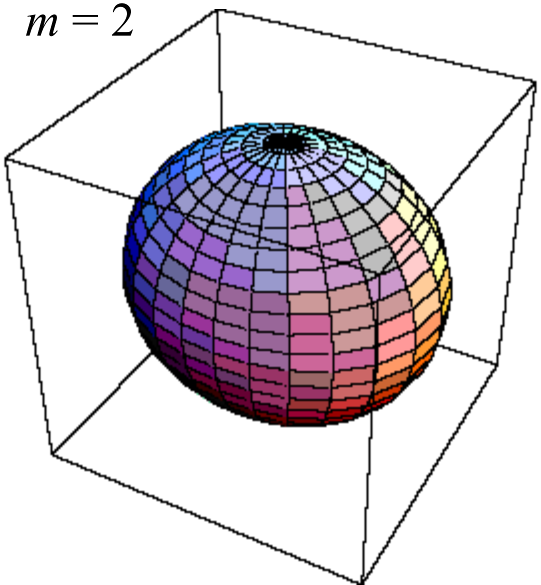


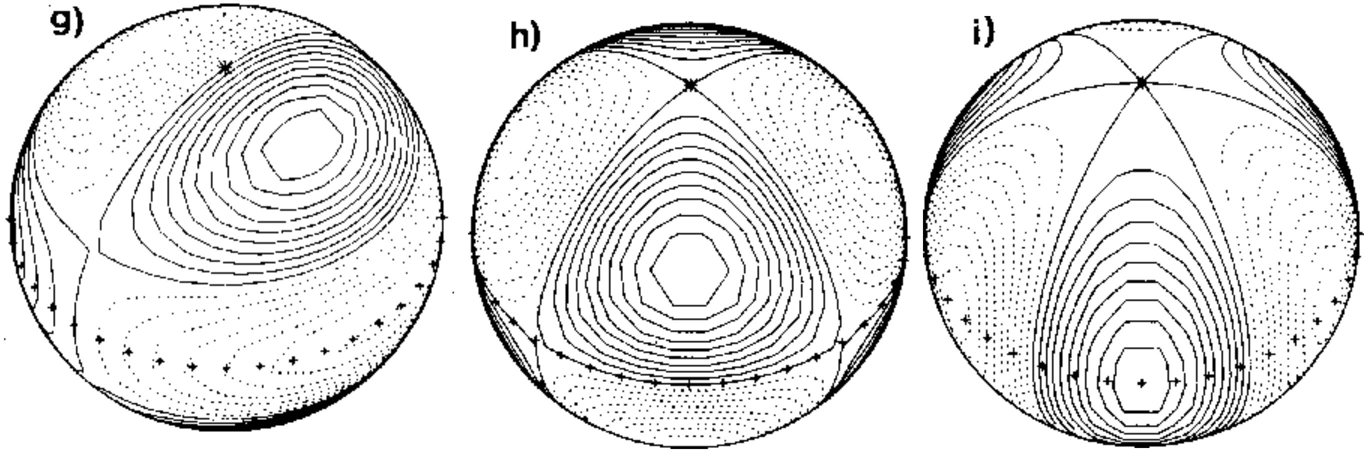
$l = 2$
$m = 1$





$l = 2$
$m = 2$



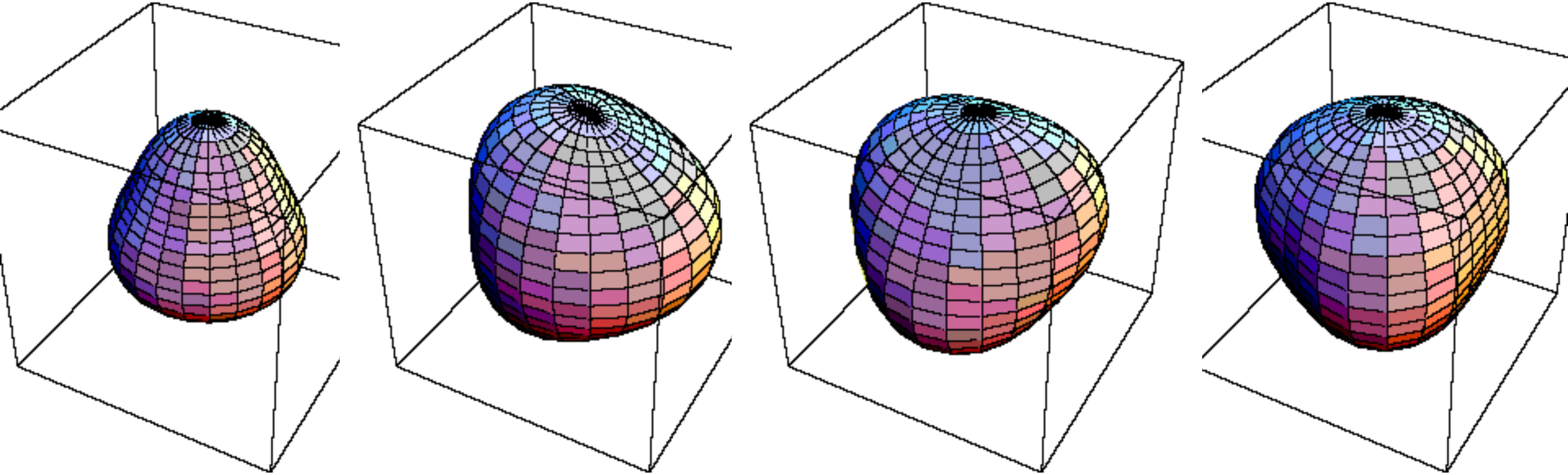


$l = 3$
$m = 0$

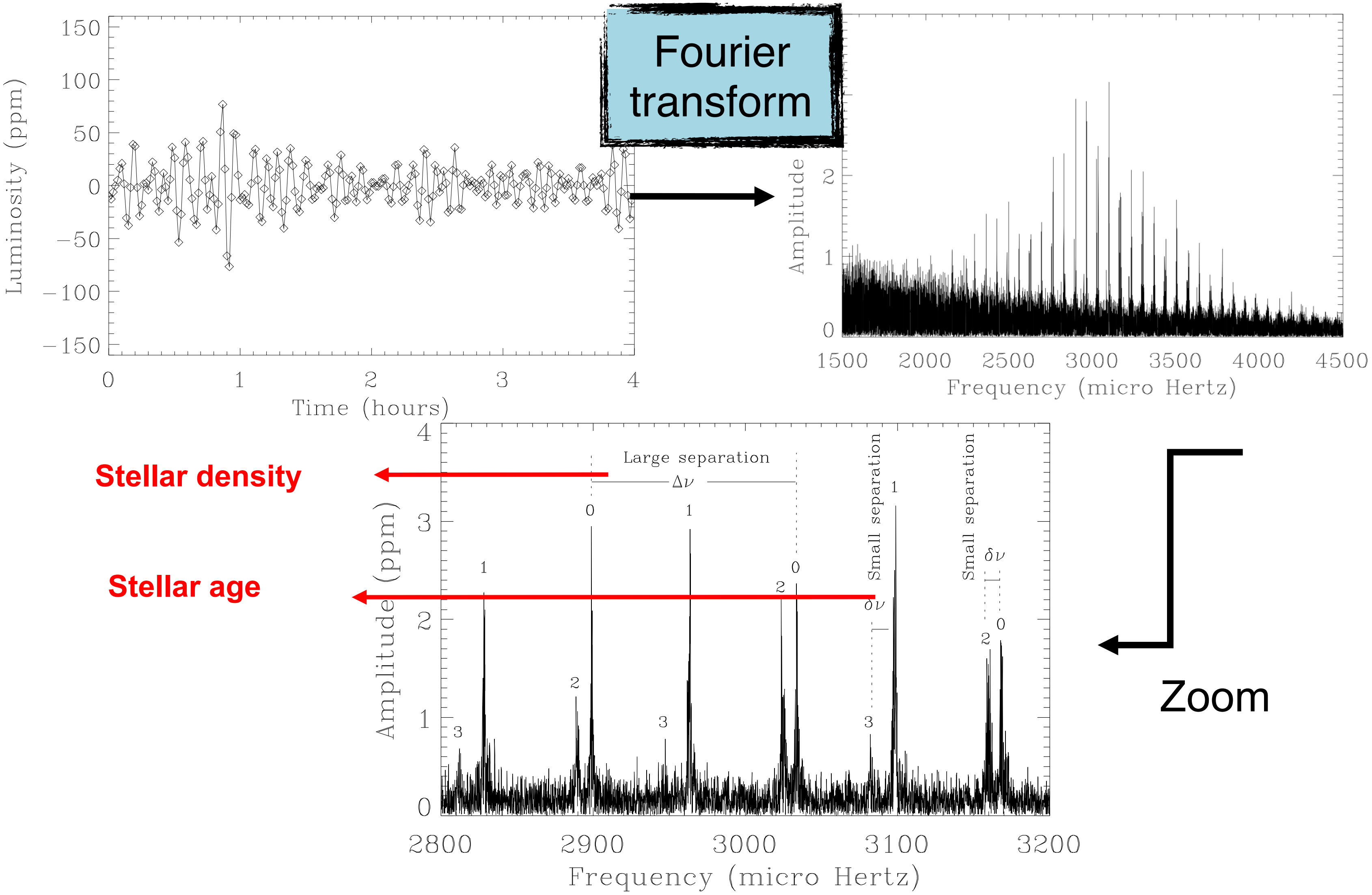
$l = 3$
$m = 1$

$l = 3$
$m = 2$

$l = 3$
$m = 3$



Asteroseismology of solar-like oscillations



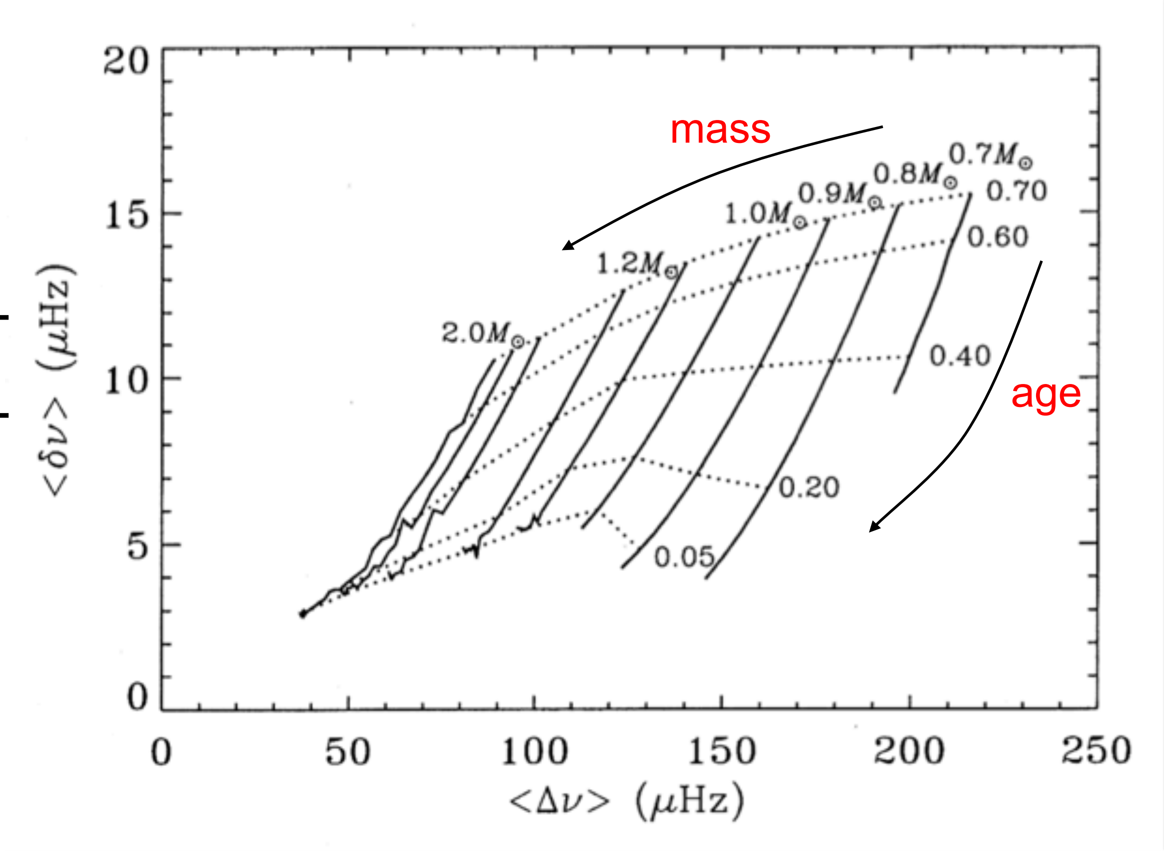
Fourier transform

Stellar density

Stellar age

Zoom

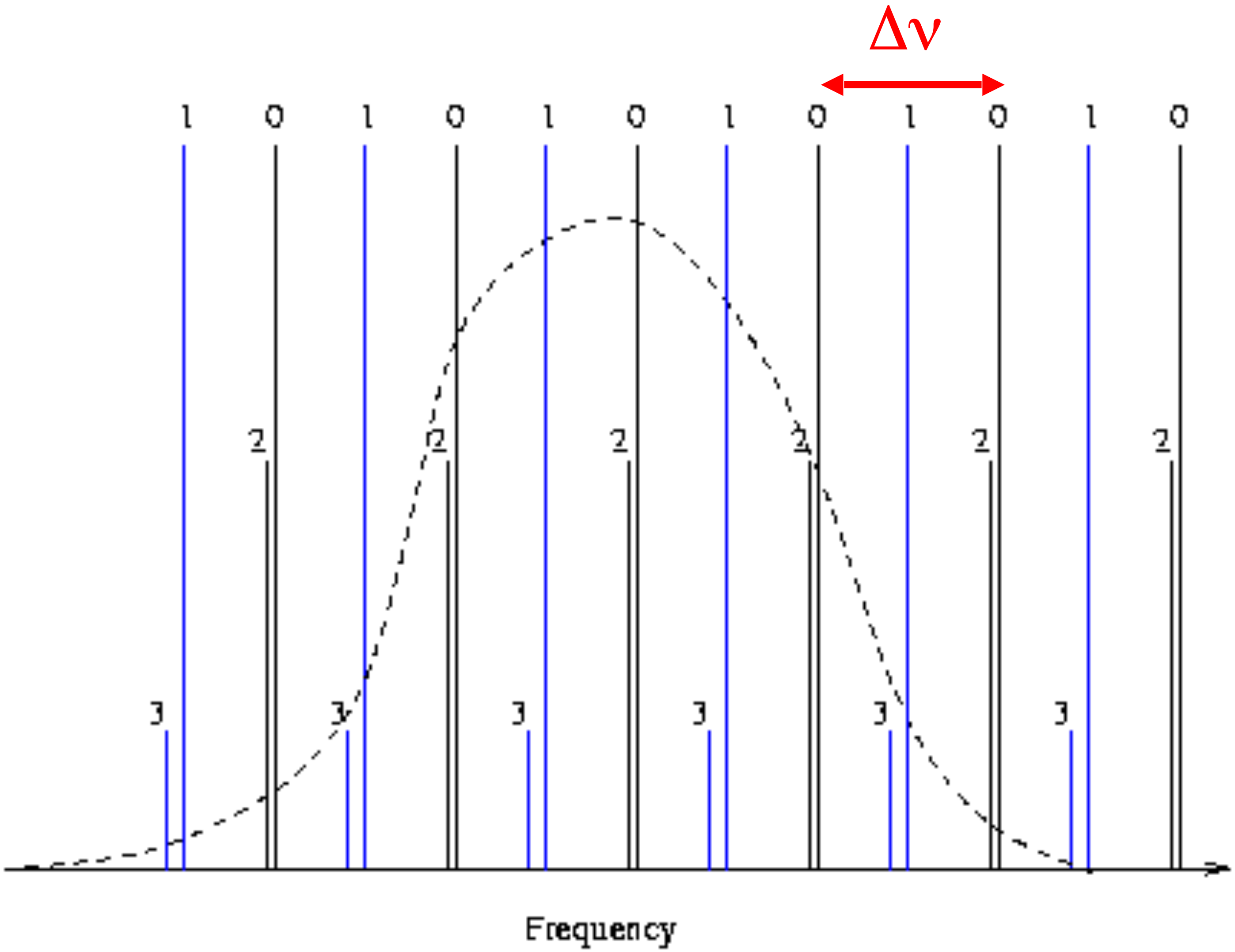
CD diagram (seismics HRD)

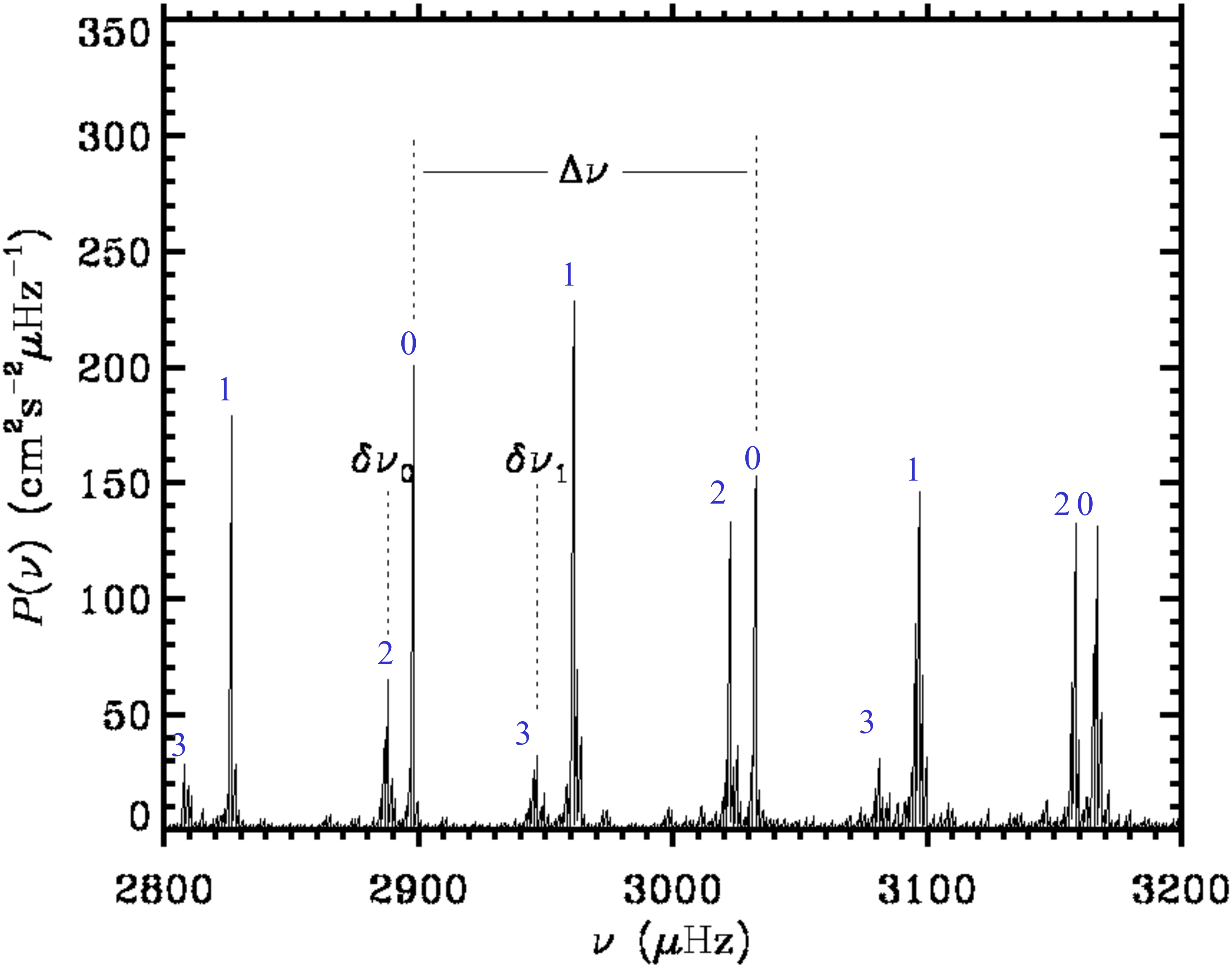


Small freq. separation

Large freq. separation

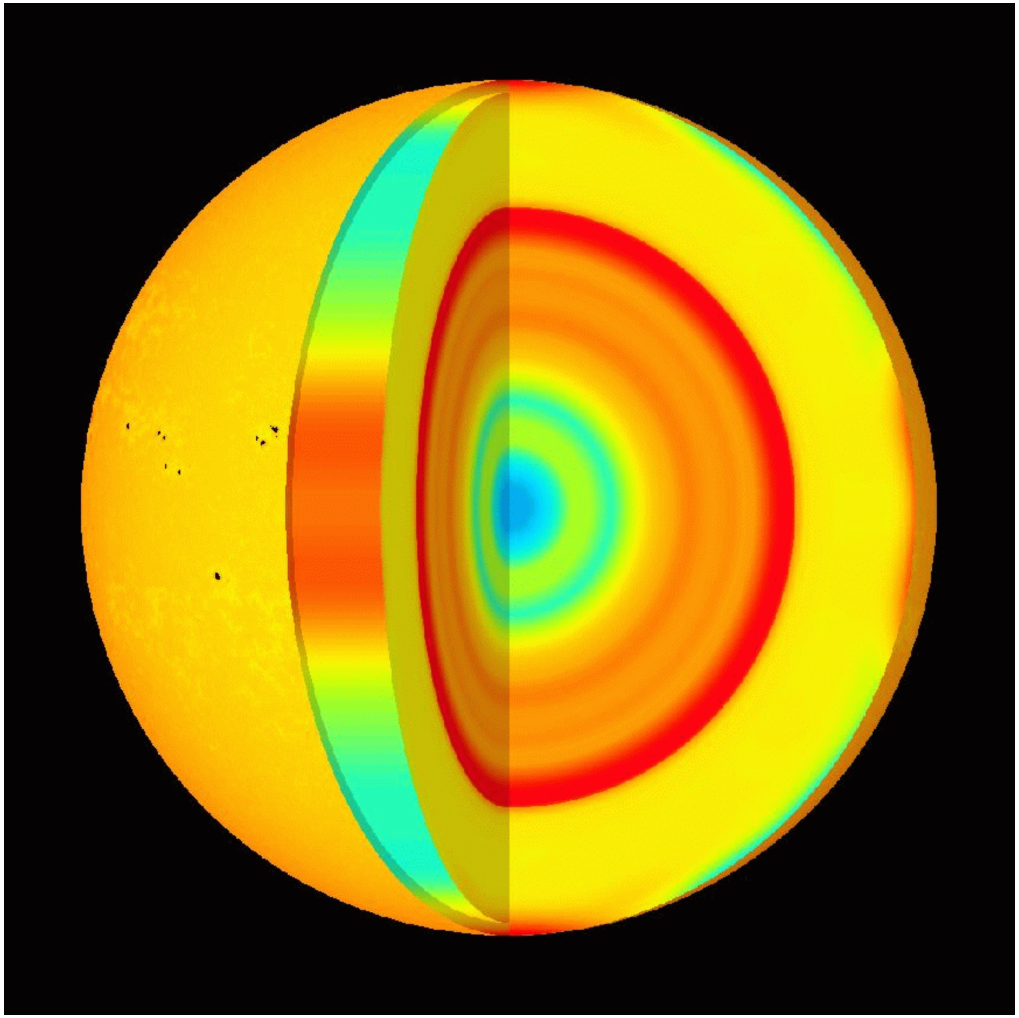
The power spectrum of solar oscillations





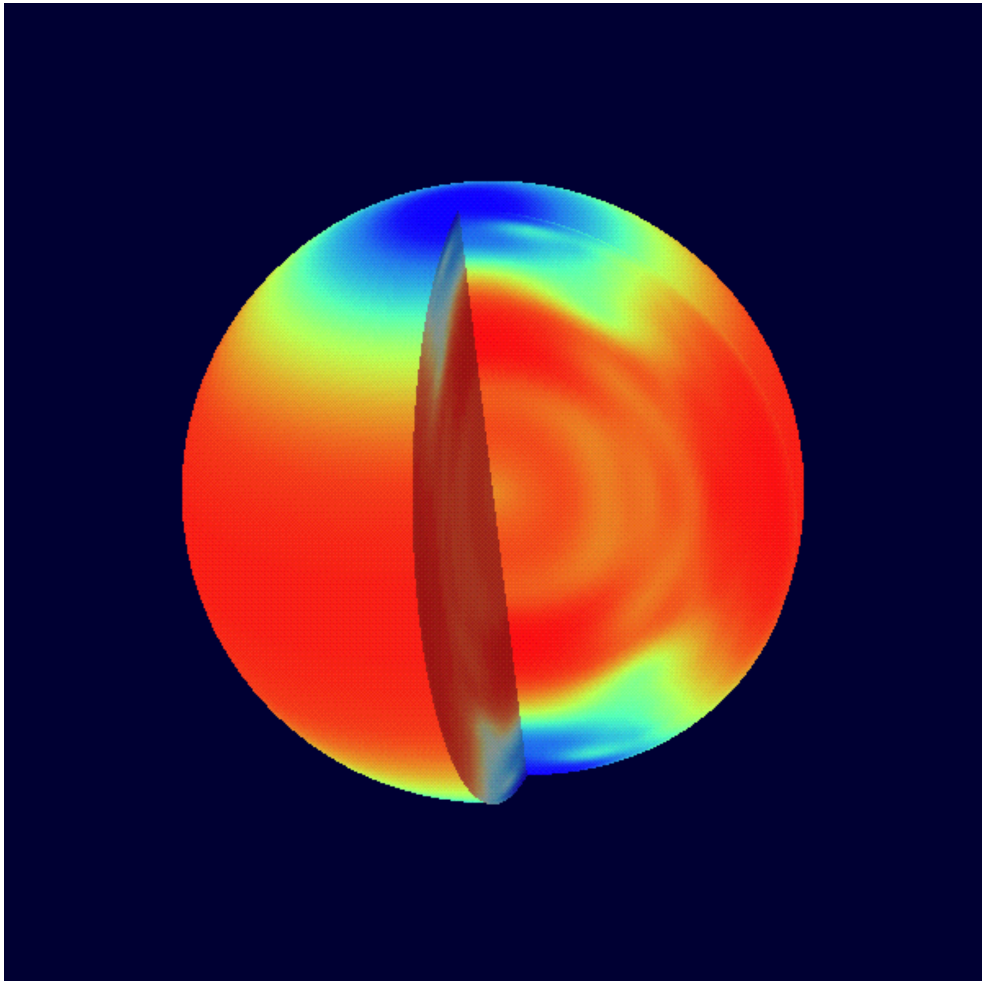
350
300
250
200
150
100
50
0
$P(\nu)$ $(\mathrm{cm}^2\mathrm{s}^{-2}\mu\mathrm{Hz}^{-1})$
2800
2900
3000
3100
3200
ν $(\mu\mathrm{Hz})$
$\Delta\nu$
$\delta\nu_0$
$\delta\nu_1$
3
1
2
0
3
1
2
0
3
1
2
0

Seismic inversion



Variations of the internal sound speed (SOHO/MDI)

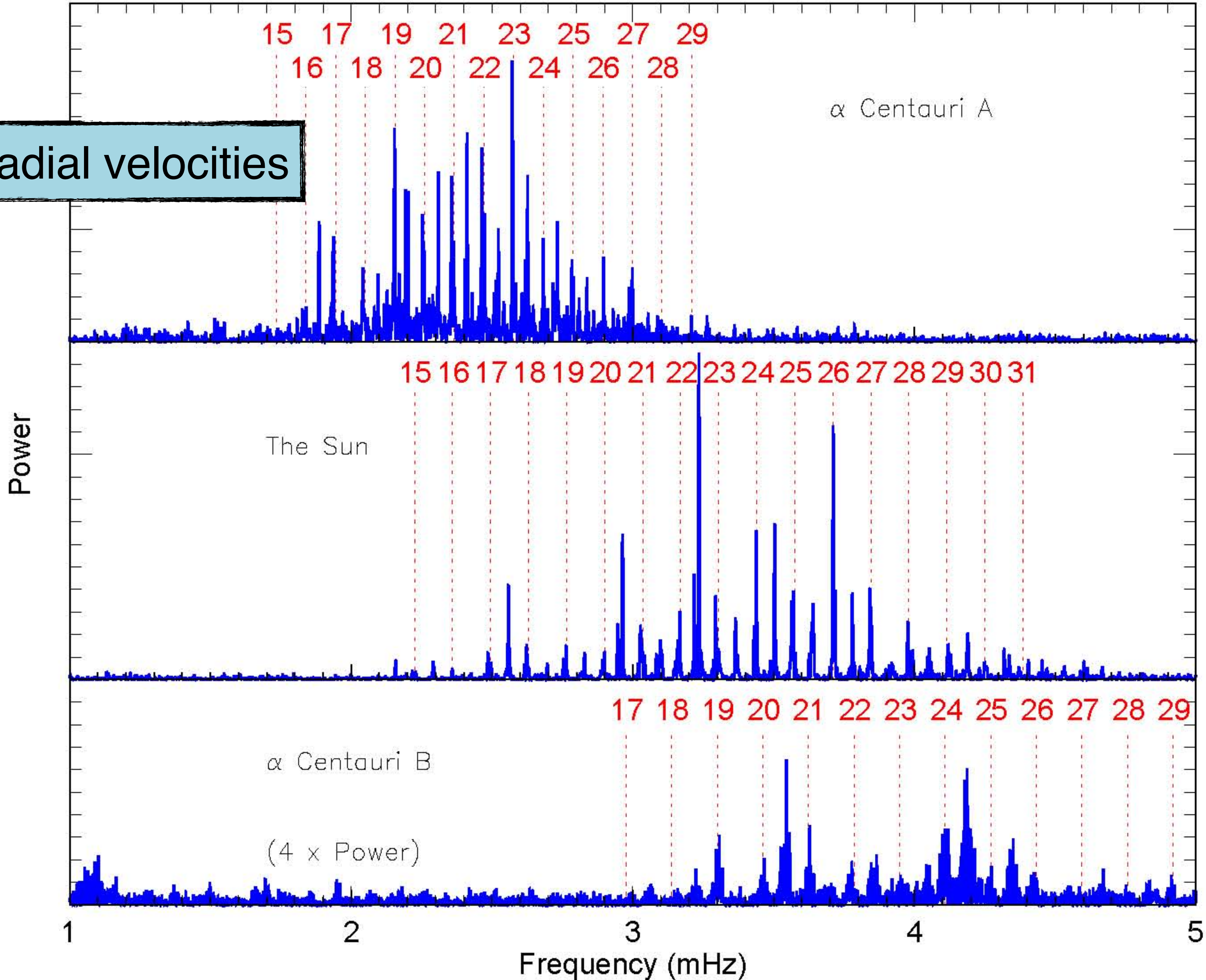
Internal rotation



(SOHO/MDI)

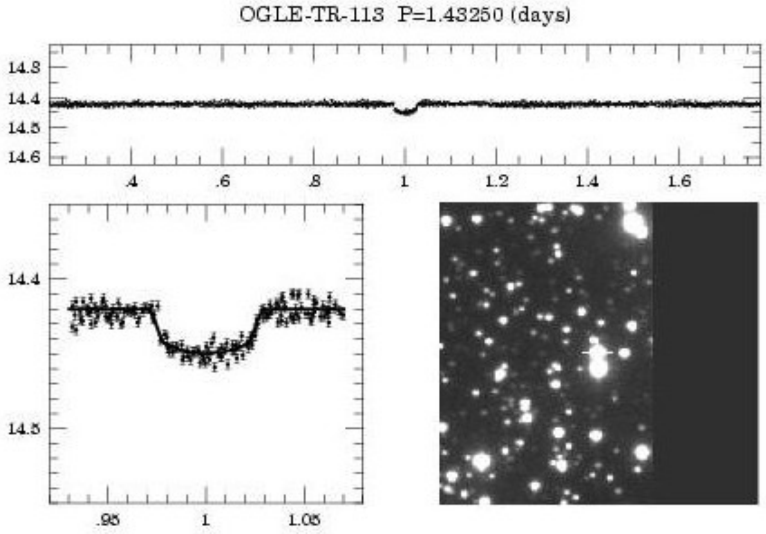
Stepping from helioseismology to asteroseismology

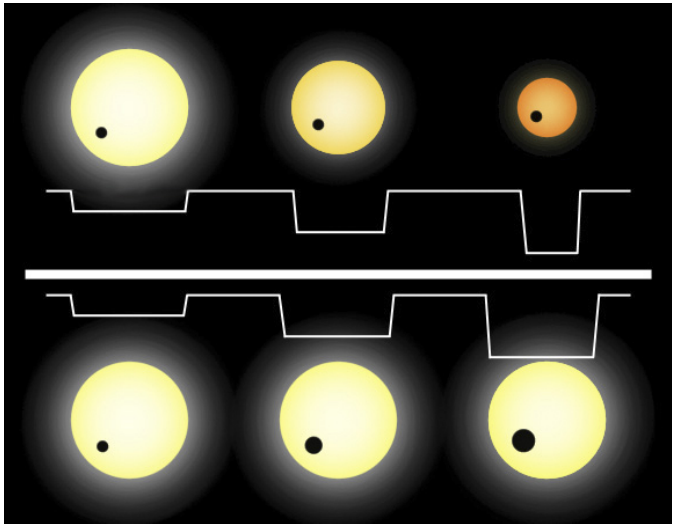
Radial velocities



Back to exoplanets (again…): transiting systems

- The planet crosses the disk of the host star
- Very important measurements!
  - planet size (w.r.t. the star)
  - density
  - **planetary structure!**
  - spectrum of planetary atmosphere
  - reflected light
  - **atmospheric structure!**
  - stellar atmosphere





Kepler

Kepler aimed at discovering Earth-like planets in the habitable zones of solar-like stars using the transit method

Uninterrupted observations of over 150,000 stars

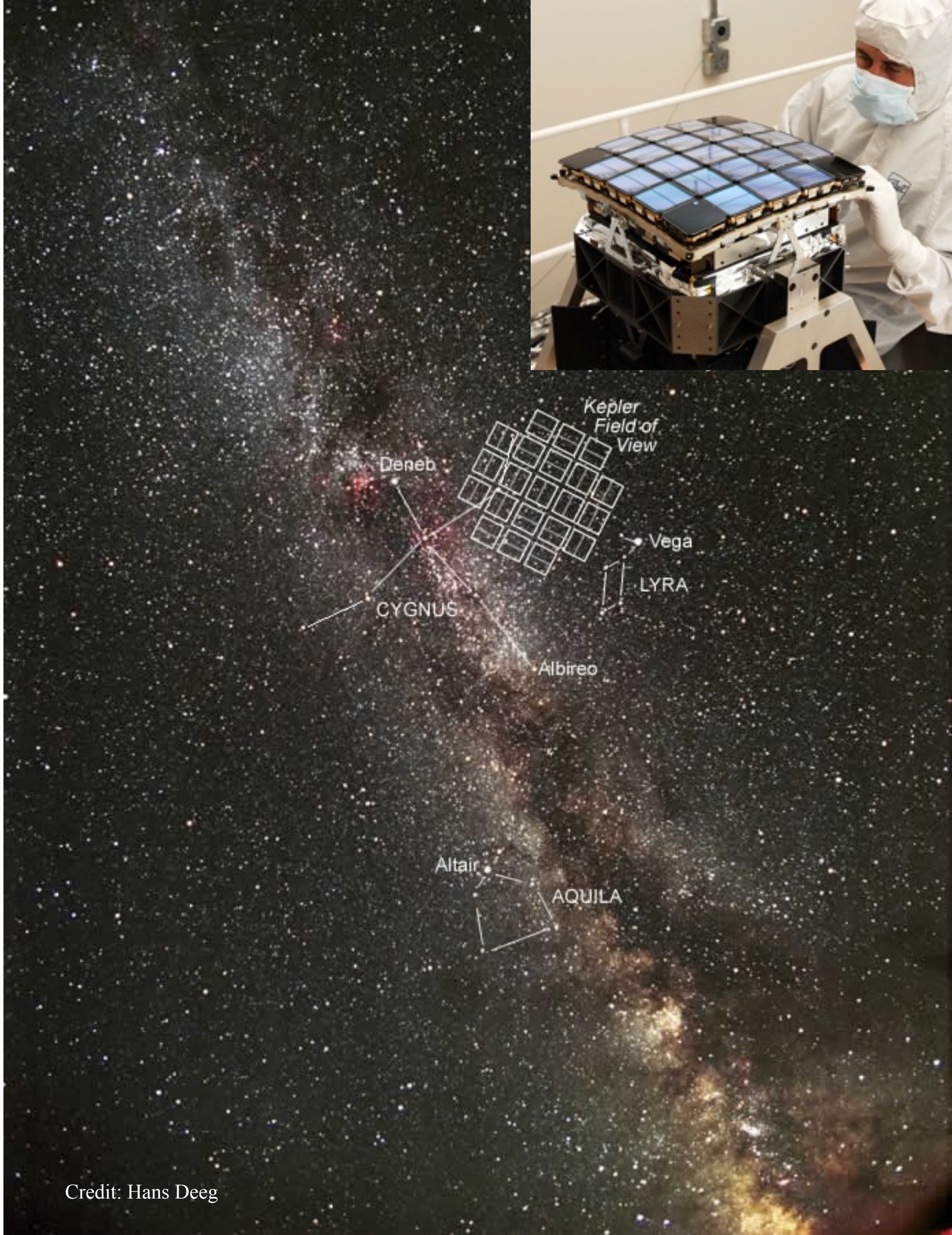
(9 < V < 15)

0.95m Schmidt telescope, 105 square degrees field of view, mosaic camera with 42 CCDs

Photometric precision:

- noise < 20 ppm after 6.5h of obs. for a 12th mag. solar-type star
- => 4-sigma detection of an exo-Earth transit

Heliocentric orbit, continuous data collecting for 4 years (2009-2013)

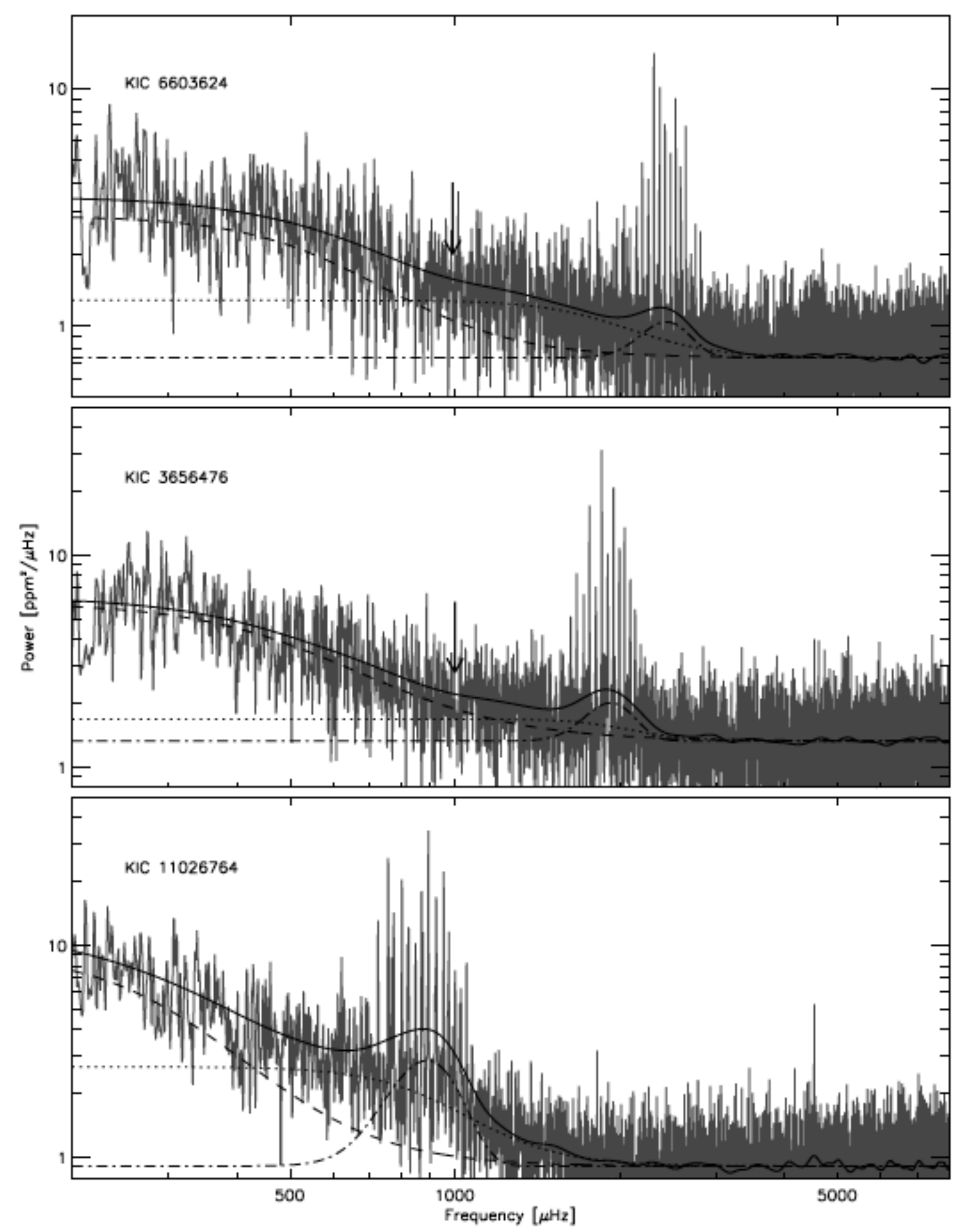


Credit: Hans Deeg

KEPLER ASTEROSEISMOLOGY

- About 4000 stars observed for Kepler Asteroseismic Science Consortium (KASC)
- Long Cadence and Short Cadence data (LC: 30 min/point; SC: 1 min/point)
- All types of stars: solar-like, white dwarfs, red giants, classical pulsators (delta Cep, RR Lyr, delta Sct, etc.), eclipsing binaries
- KASC: >500 scientists from >25 countries
- 14 working groups, two led by Konkoly astronomers (Róbert Szabó, László Kiss)

Solar-like stars



Chaplin et al. (2010)

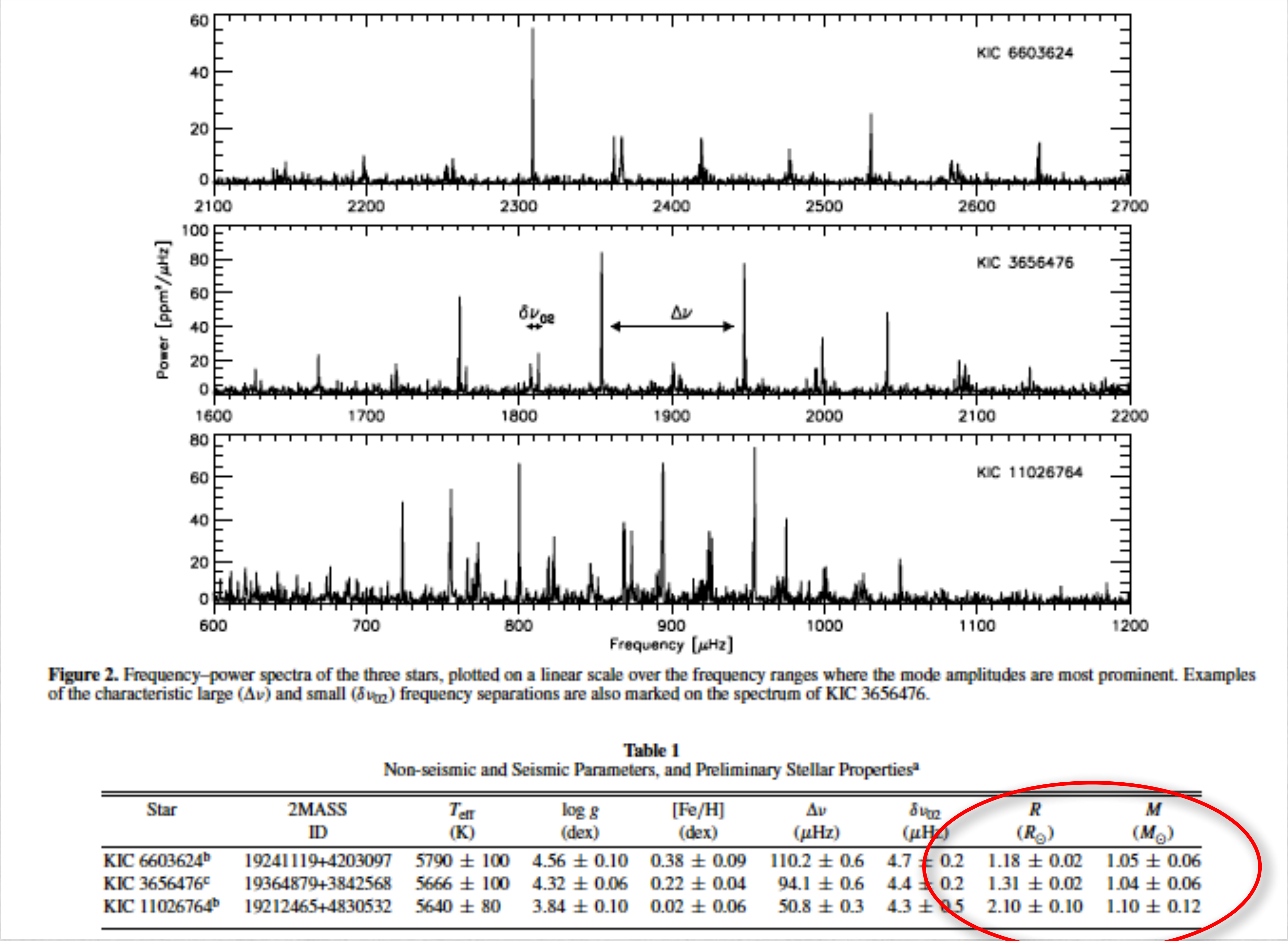
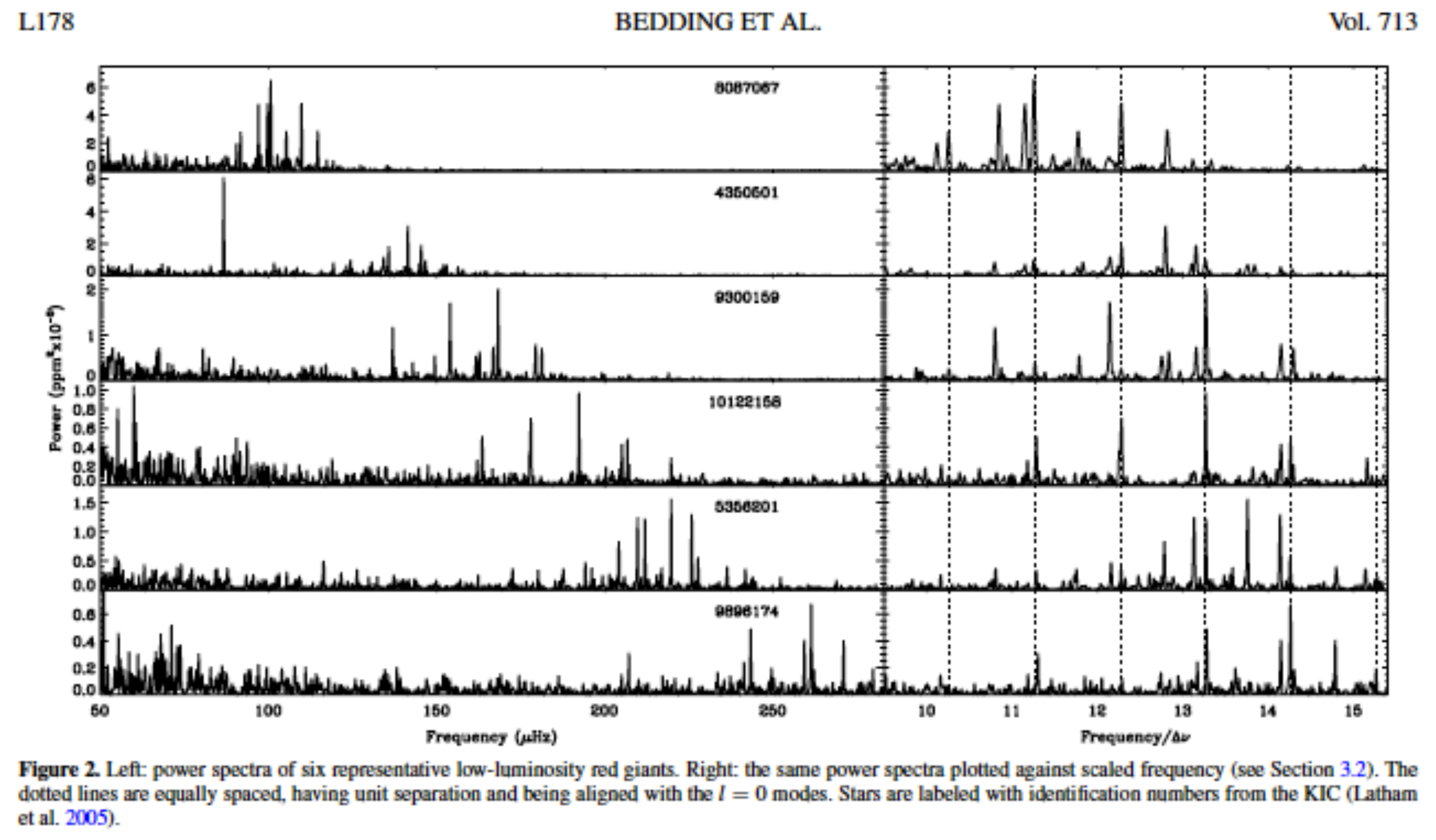


Figure 2. Frequency–power spectra of the three stars, plotted on a linear scale over the frequency ranges where the mode amplitudes are most prominent. Examples of the characteristic large ($\Delta\nu$) and small ($\delta\nu_{02}$) frequency separations are also marked on the spectrum of KIC 3656476.

Table 1
Non-seismic and Seismic Parameters, and Preliminary Stellar Properties[a]

Star	2MASS ID	$T_{\rm eff}$ (K)	log g (dex)	[Fe/H] (dex)	$\Delta\nu$ (μHz)	$\delta\nu_{02}$ (μHz)	R ($R_\odot$)	M ($M_\odot$)
KIC 6603624[b]	19241119+4203097	5790 ± 100	4.56 ± 0.10	0.38 ± 0.09	110.2 ± 0.6	4.7 ± 0.2	1.18 ± 0.02	1.05 ± 0.06
KIC 3656476[c]	19364879+3842568	5666 ± 100	4.32 ± 0.06	0.22 ± 0.04	94.1 ± 0.6	4.4 ± 0.2	1.31 ± 0.02	1.04 ± 0.06
KIC 11026764[b]	19212465+4830532	5640 ± 80	3.84 ± 0.10	0.02 ± 0.06	50.8 ± 0.3	4.3 ± 0.5	2.10 ± 0.10	1.10 ± 0.12

Low luminosity red giants: solar-like oscillations are ubiquitous



L178 BEDDING ET AL. Vol. 713

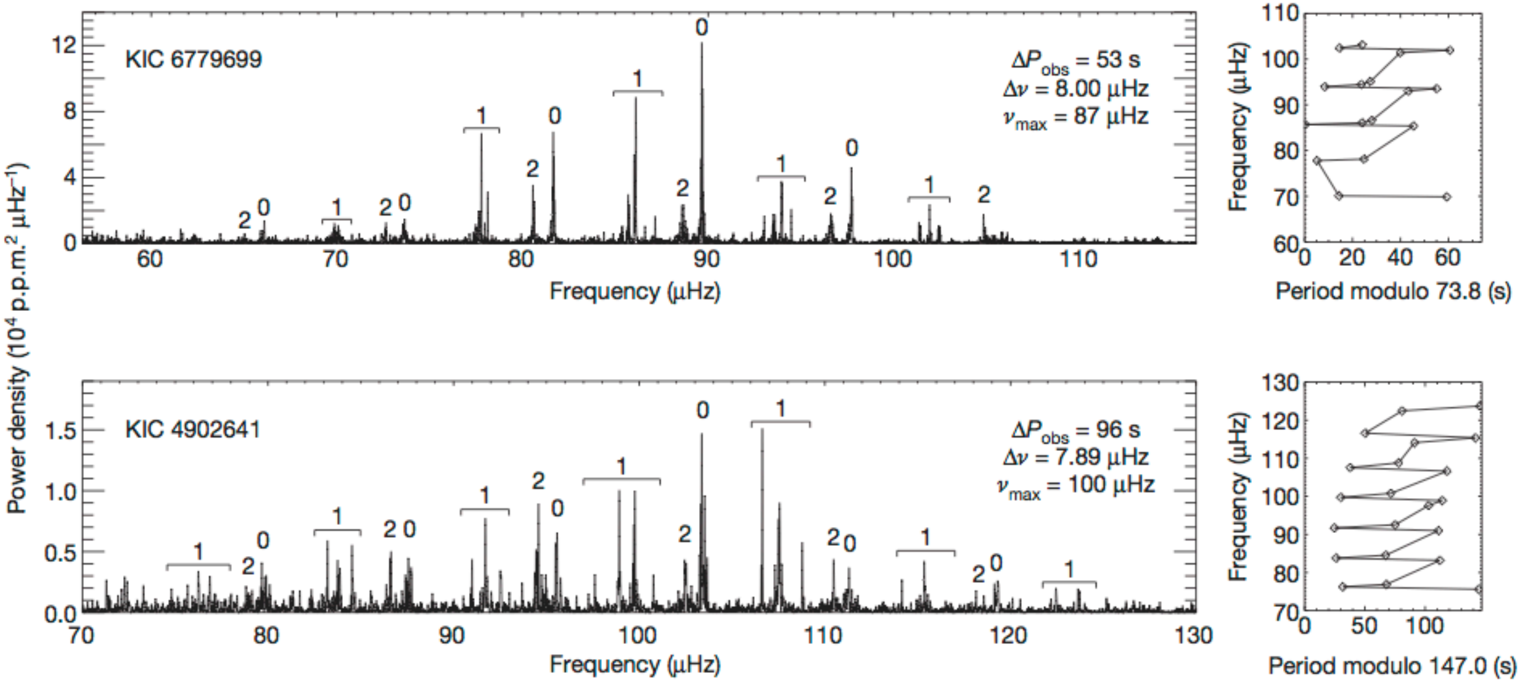
Figure 2. Left: power spectra of six representative low-luminosity red giants. Right: the same power spectra plotted against scaled frequency (see Section 3.2). The dotted lines are equally spaced, having unit separation and being aligned with the $l = 0$ modes. Stars are labeled with identification numbers from the KIC (Latham et al. 2005).

LETTER

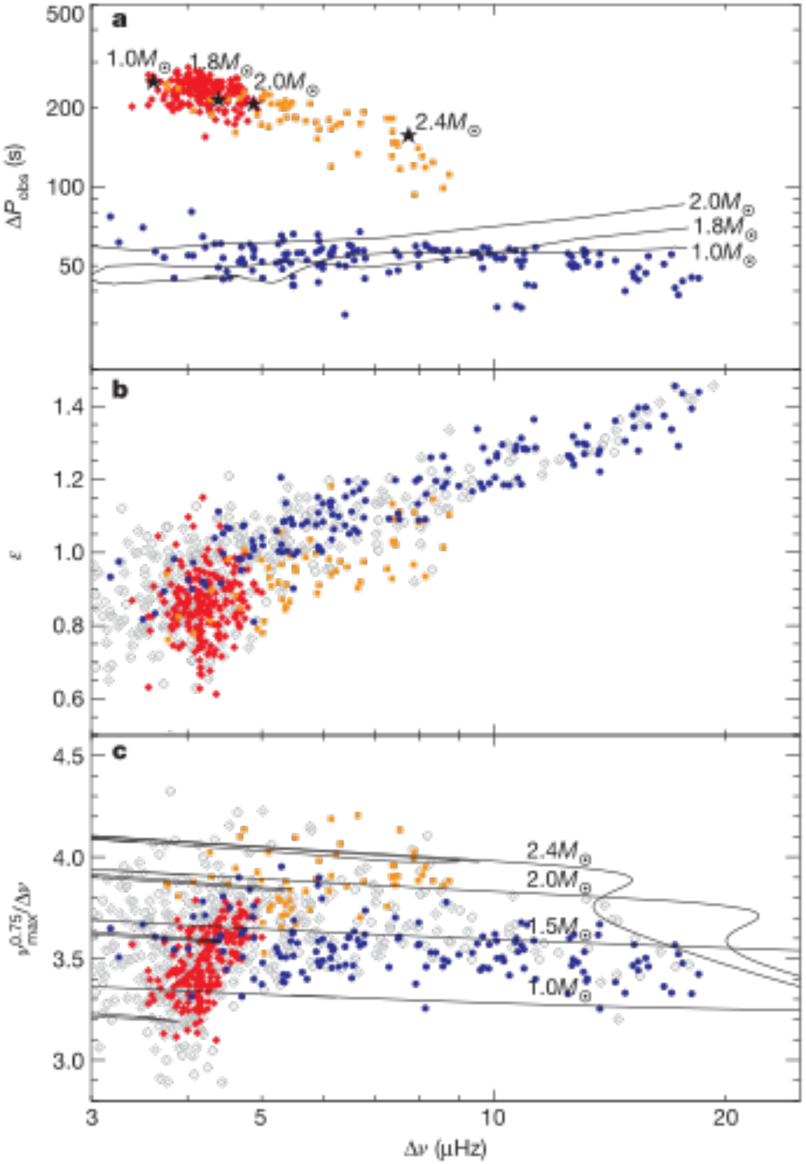
doi:10.1038/nature09935

Gravity modes as a way to distinguish between hydrogen- and helium-burning red giant stars

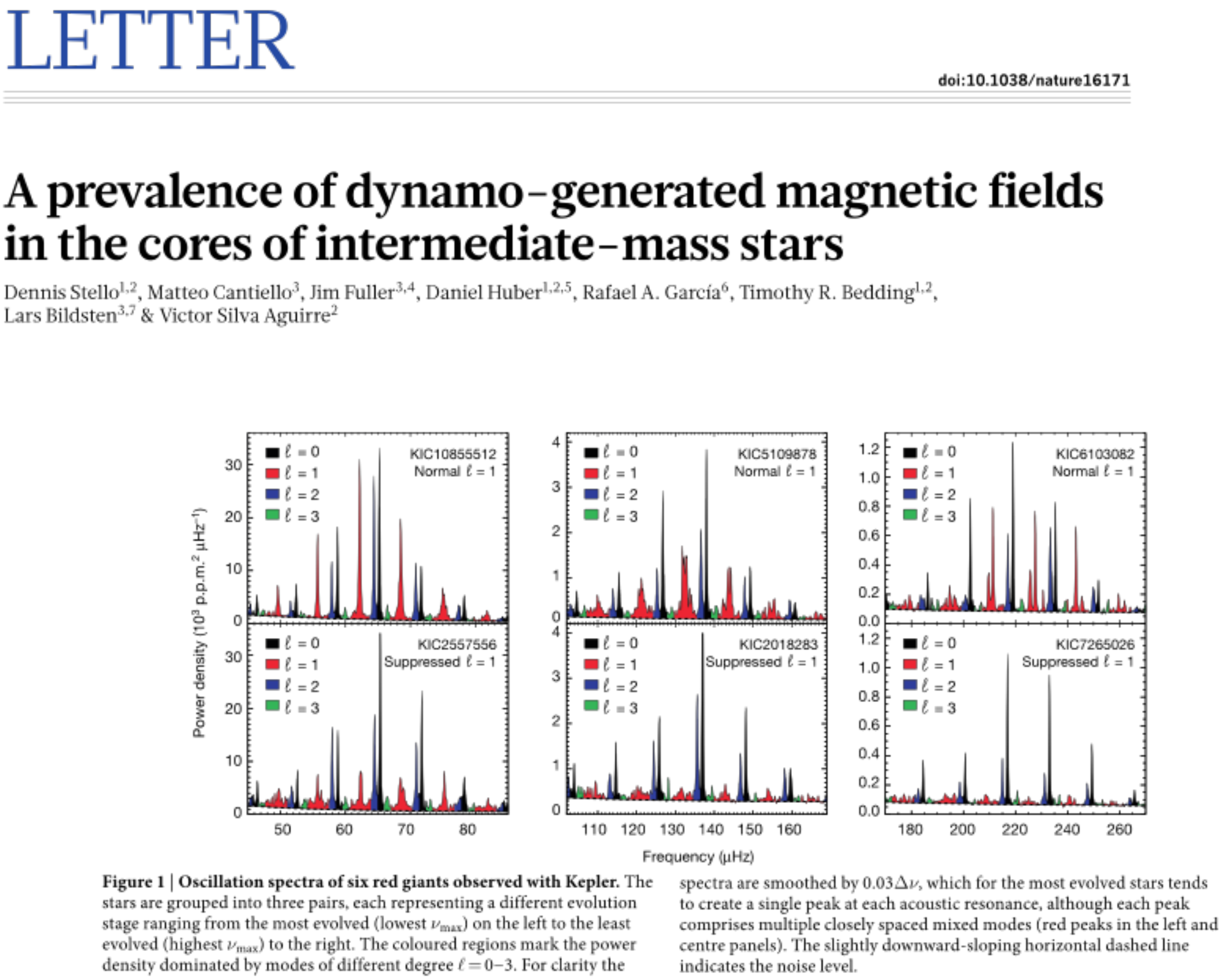
Timothy R. Bedding[1], Benoit Mosser[2], Daniel Huber[1], Josefina Montalbán[3], Paul Beck[4], Jørgen Christensen-Dalsgaard[5], Yvonne P. Elsworth[6], Rafael A. García[7], Andrea Miglio[3,6], Dennis Stello[1], Timothy R. White[1], Joris De Ridder[4], Saskia Hekker[6,8], Conny Aerts[4,9], Caroline Barban[2], Kevin Belkacem[10], Anne-Marie Broomhall[6], Timothy M. Brown[11], Derek L. Buzasi[12], Fabien Carrier[4], William J. Chaplin[6], Maria Pia Di Mauro[13], Marc-Antoine Dupret[3], Søren Frandsen[5], Ronald L. Gilliland[14], Marie-Jo Goupil[2], Jon M. Jenkins[15], Thomas Kallinger[16], Steven Kawaler[17], Hans Kjeldsen[5], Savita Mathur[18], Arlette Noels[3], Victor Silva Aguirre[19] & Paolo Ventura[20]



Bedding et al., Nature, 2011 March 11



Bedding et al., Nature, 2011 March 11



LETTER

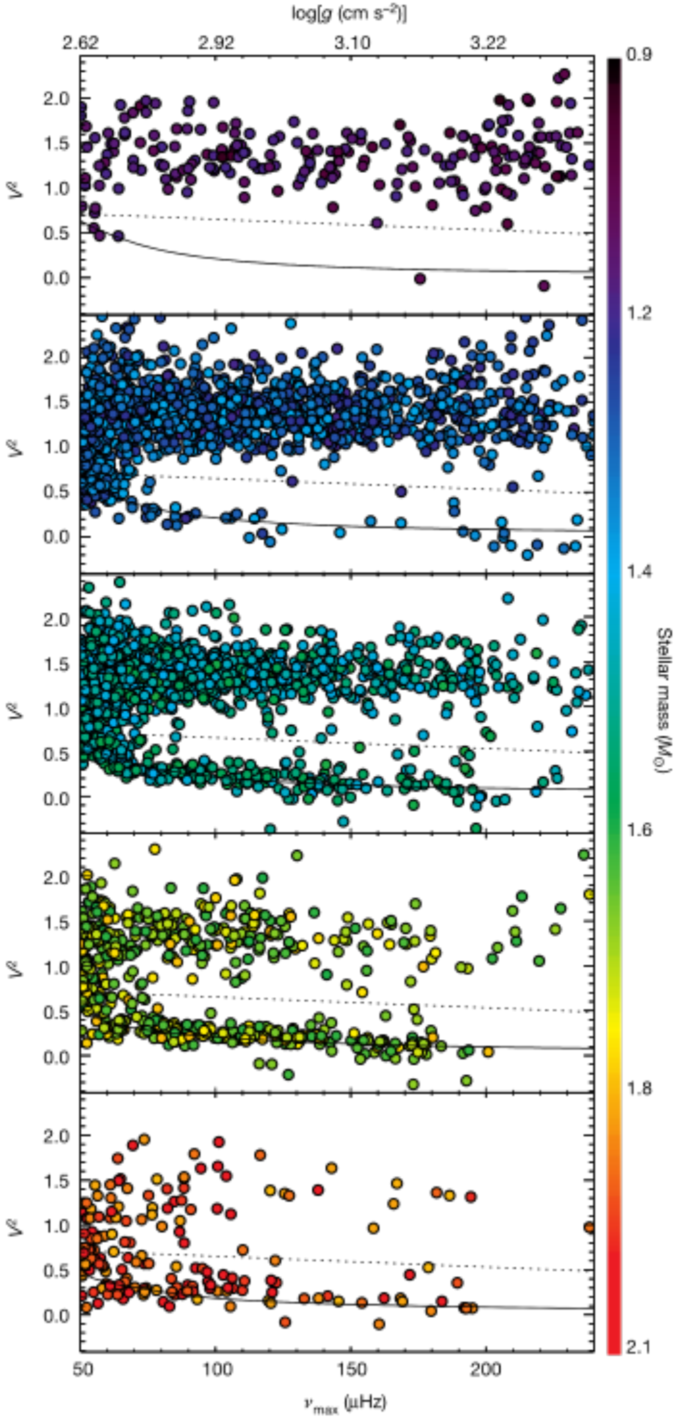
doi:10.1038/nature16171

A prevalence of dynamo-generated magnetic fields in the cores of intermediate-mass stars

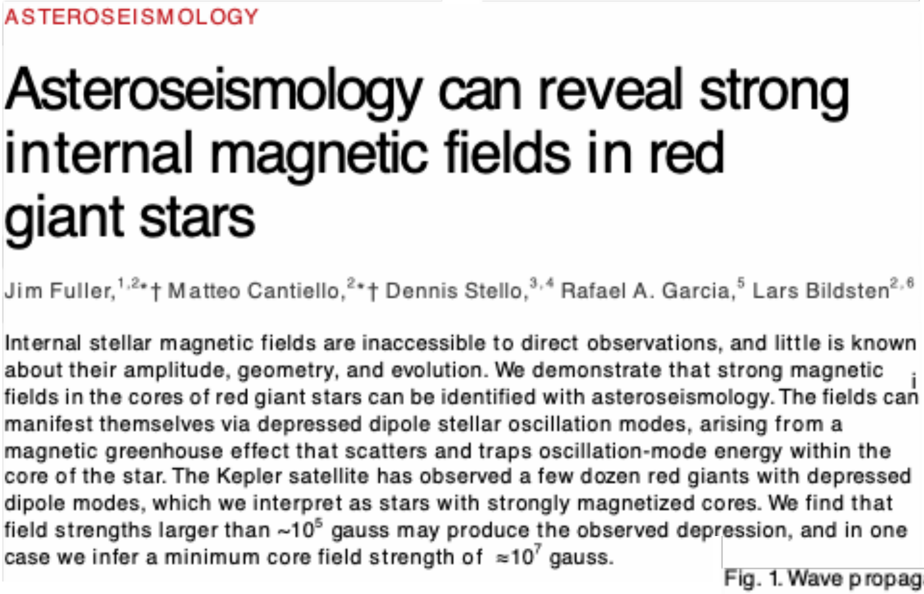
Dennis Stello[1,2], Matteo Cantiello[3], Jim Fuller[3,4], Daniel Huber[1,2,5], Rafael A. García[6], Timothy R. Bedding[1,2], Lars Bildsten[3,7] & Victor Silva Aguirre[2]

Figure 1 | Oscillation spectra of six red giants observed with Kepler. The stars are grouped into three pairs, each representing a different evolution stage ranging from the most evolved (lowest $\nu_{\max}$) on the left to the least evolved (highest $\nu_{\max}$) to the right. The coloured regions mark the power density dominated by modes of different degree $\ell = 0$–3. For clarity the spectra are smoothed by $0.03\Delta\nu$, which for the most evolved stars tends to create a single peak at each acoustic resonance, although each peak comprises multiple closely spaced mixed modes (red peaks in the left and centre panels). The slightly downward-sloping horizontal dashed line indicates the noise level.

Stello et al., Nature, 2016 January 21



Stello et al., Nature, 2016 January 21



ASTEROSEISMOLOGY

Asteroseismology can reveal strong internal magnetic fields in red giant stars

Jim Fuller,[1,2]*† Matteo Cantiello,[2]*† Dennis Stello,[3,4] Rafael A. Garcia,[5] Lars Bildsten[2,6]

Internal stellar magnetic fields are inaccessible to direct observations, and little is known about their amplitude, geometry, and evolution. We demonstrate that strong magnetic fields in the cores of red giant stars can be identified with asteroseismology. The fields can manifest themselves via depressed dipole stellar oscillation modes, arising from a magnetic greenhouse effect that scatters and traps oscillation-mode energy within the core of the star. The Kepler satellite has observed a few dozen red giants with depressed dipole modes, which we interpret as stars with strongly magnetized cores. We find that field strengths larger than ~10^5 gauss may produce the observed depression, and in one case we infer a minimum core field strength of ≈10^7 gauss.

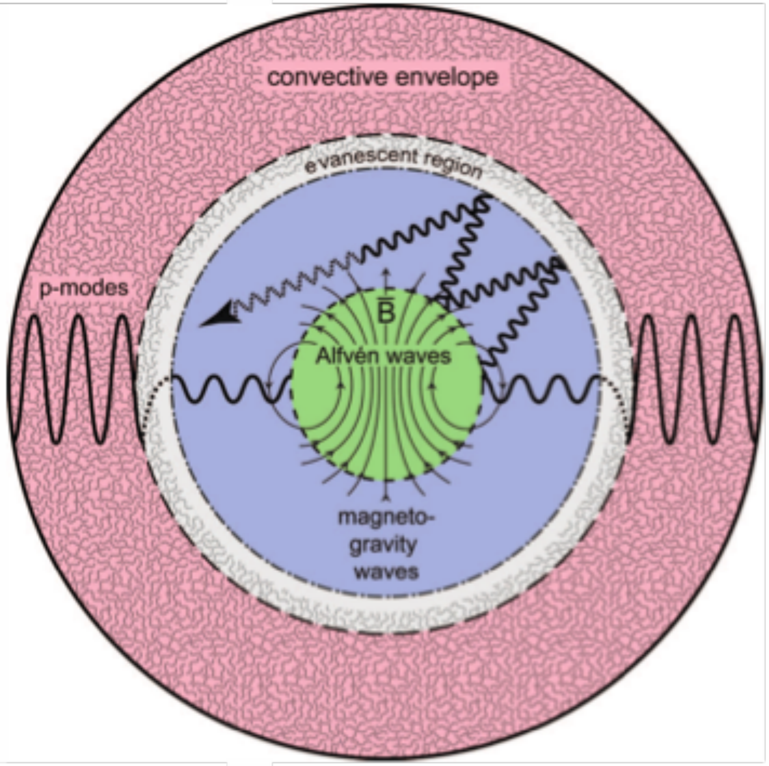


Fig. 1. Wave propagation in red giants with magnetized cores. Acoustic waves excited in the envelope couple to gravity waves in the radiative core. In the presence of a magnetic field in the core, the gravity waves are scattered at regions of high field strength. Because the field cannot be spherically symmetric, the waves are scattered to high angular degree ℓ and become trapped within the core, where they eventually dissipate (dashed wave with arrow). We refer to this as the magnetic greenhouse effect.

Fuller et al., Science, 2015 October 23

The non-linear regime: chaos in astrophysics

When Fourier analysis just fails: some stars seem to be truly irregular

1. Is it possible to define the regularity or irregularity?

2. Since the signal is our main source of information, can the irregularity of the signal tell us something about its origins?

3. How many coupled variables are needed to model the irregular dynamics?

Chaos: a simplistic summary

- low-order deterministic system with “erratic” behaviour
- exponential blow-up of the trajectories in the phase space
  → Lyapunov-exponents (Lyapunov-spectrum)
- phase space reconstruction can be done with time-delay embedding (Takens theorem)

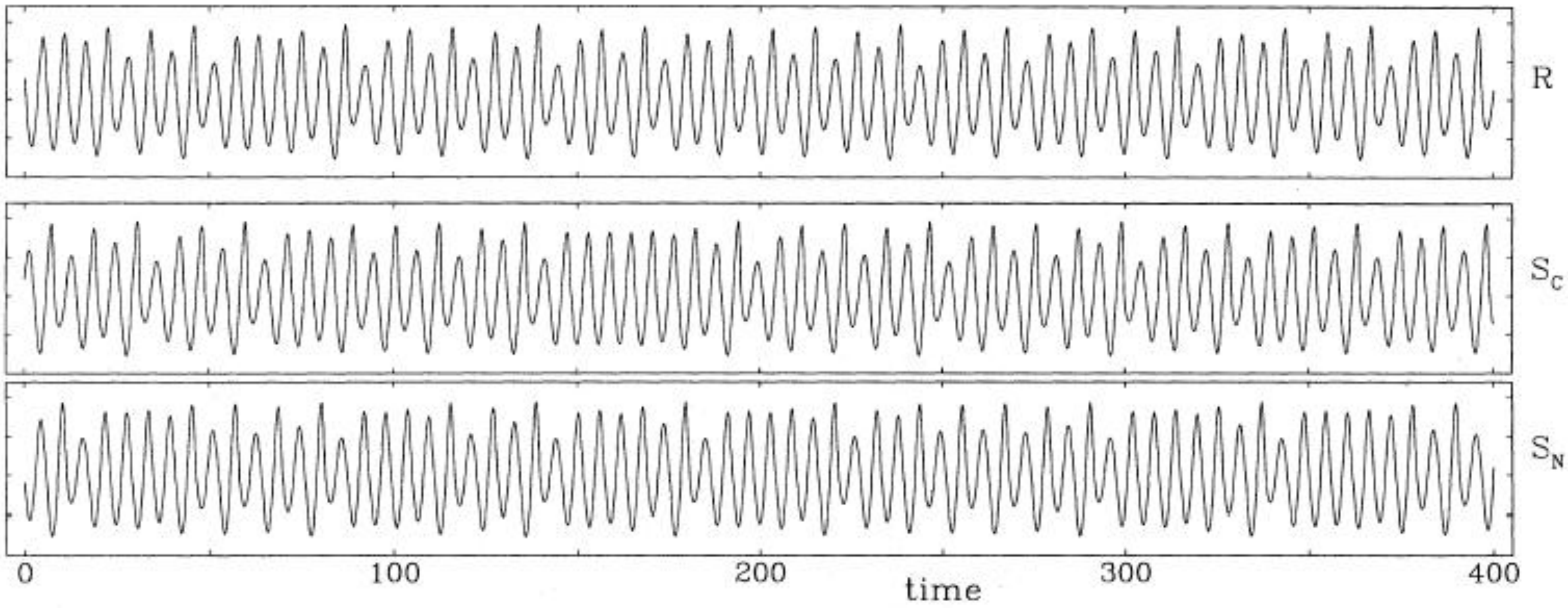
A well-studied example: the Rössler oscillator

$$\frac{dx_1}{dt} = -x_2 - x_3$$

$$\frac{dx_2}{dt} = x_1 + ax_2$$

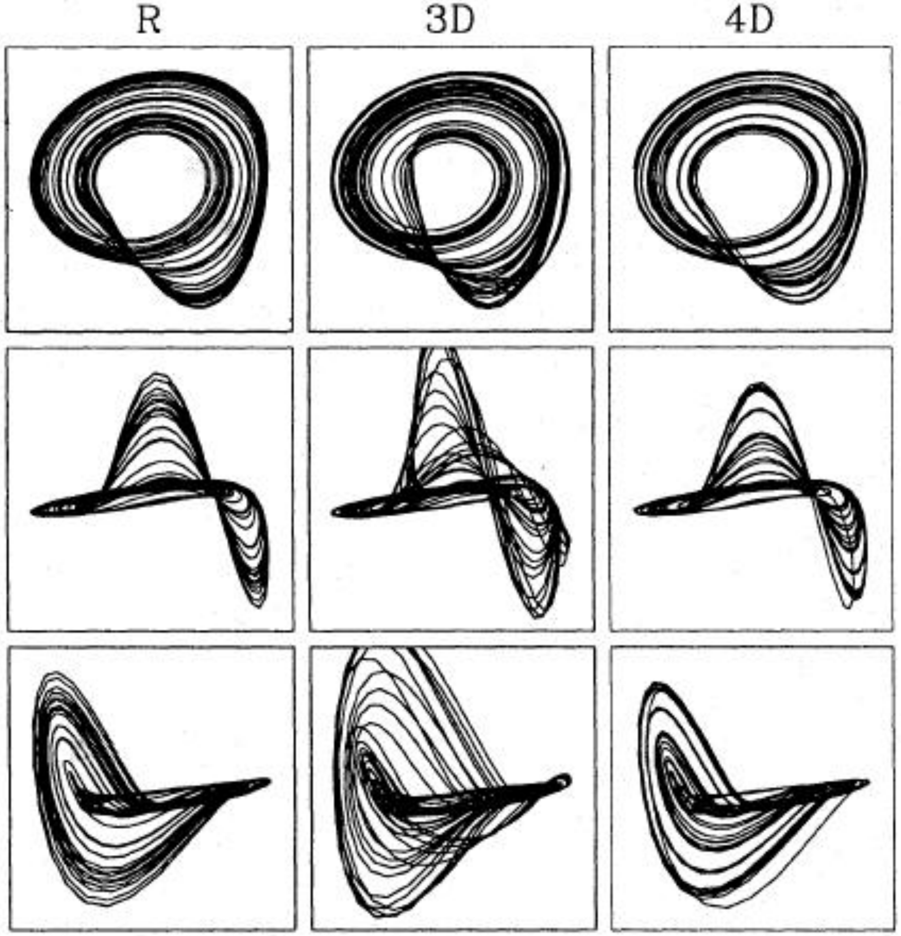
$$\frac{dx_3}{dt} = b + (x_1 - c)x_3$$

Rössler data in time domain:



Serre et al. (1996)

Phase space projections:



Serre et al. (1996)

Problem: it needs looooooooooong time series!

A brief history of chaos in astrophysics

Late 70's: chaos is “discovered” by physicists
One of the first astronomical applications:
Blacher & Perdang (1981): the phase coherence of solar oscillations does not necessarily support the normal mode interpretation

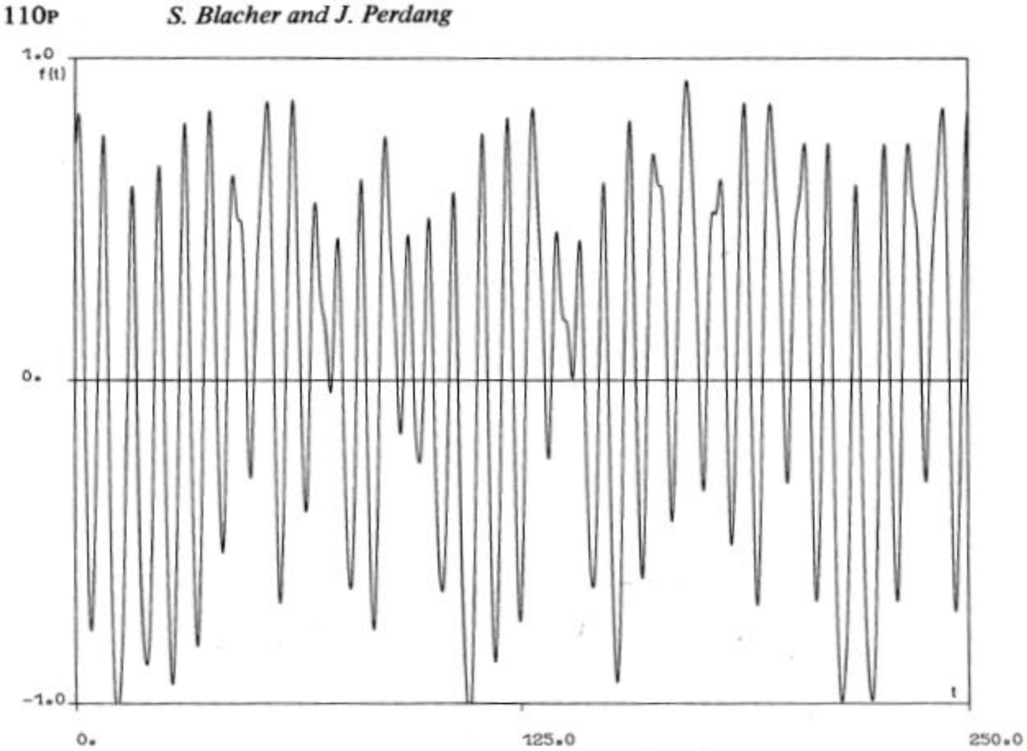

110P S. Blacher and J. Perdang

Figure 1. Time-behaviour of the chaotic function $f(t)$ (equation 2).

Primary targets for chaos studies in astrophysics:

complex systems in *celestial mechanics*
(complicated gravitational interactions)
Solar System, exoplanets around other stars,
stellar orbits in galactic potentials

hydrodynamic phenomena in stars

→ *stellar oscillations*

The first giant star with chaotic pulsations: R Scuti
(Buchler et al. 1996)

5.5 years of data (130 yr in total) :

No. 1, 1996 NONLINEAR ANALYSIS OF R SCUTI 491

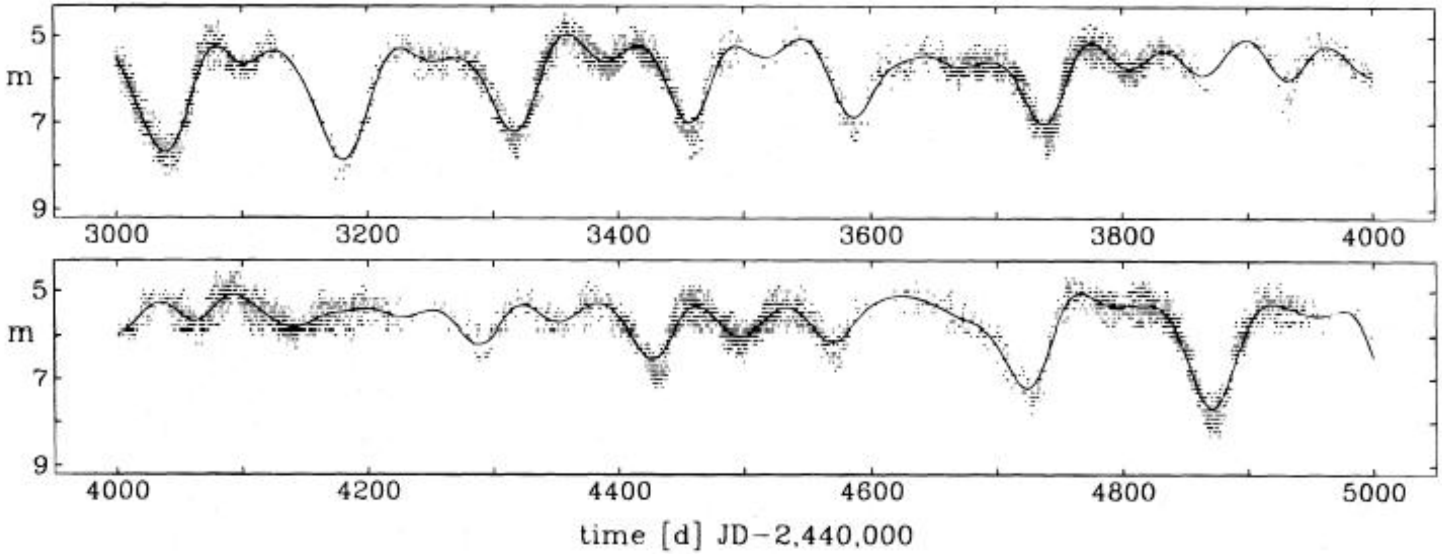
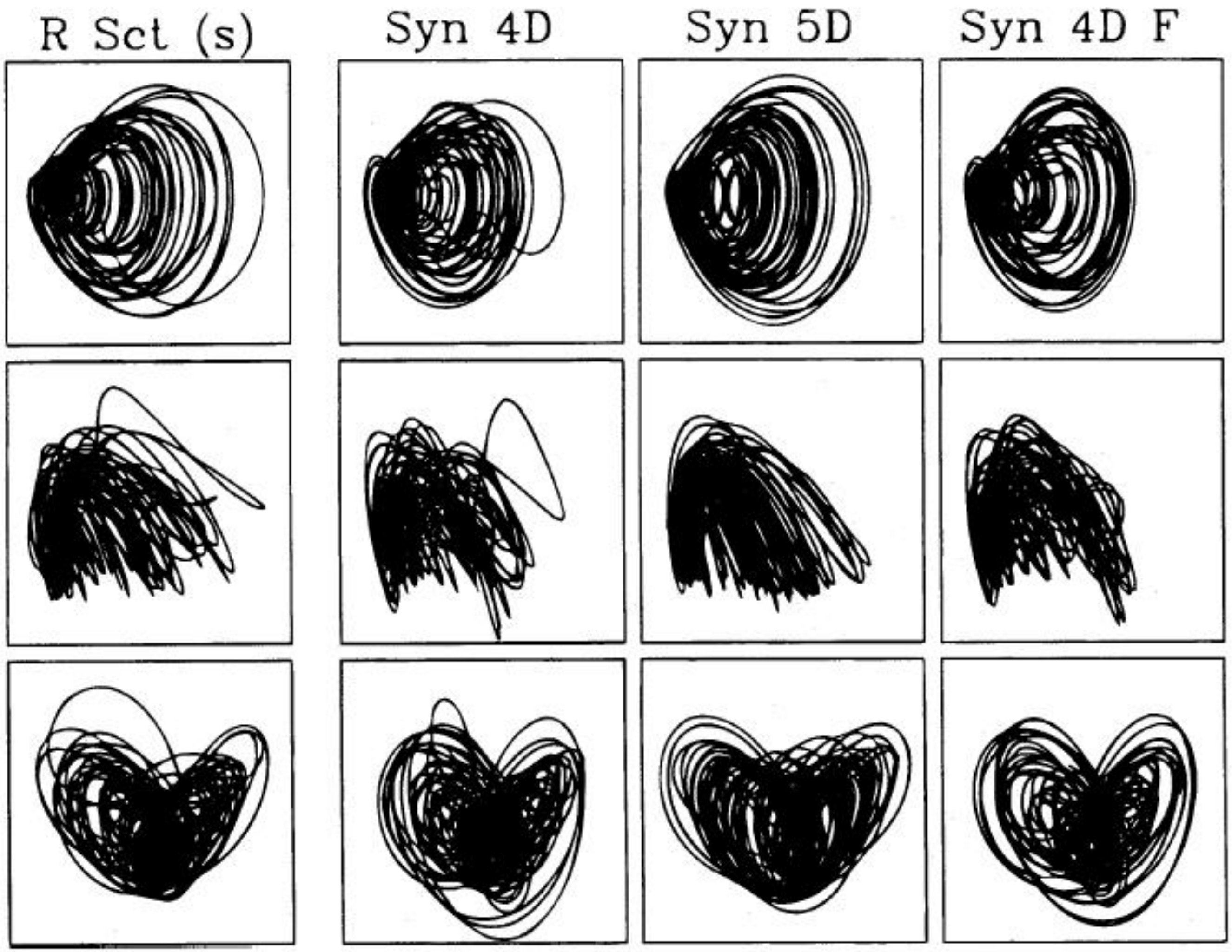


FIG. 1.—Typical observed light curve segments for R Scuti. *Dots:* the individual observations; *line:* the smoothed filtered signal.

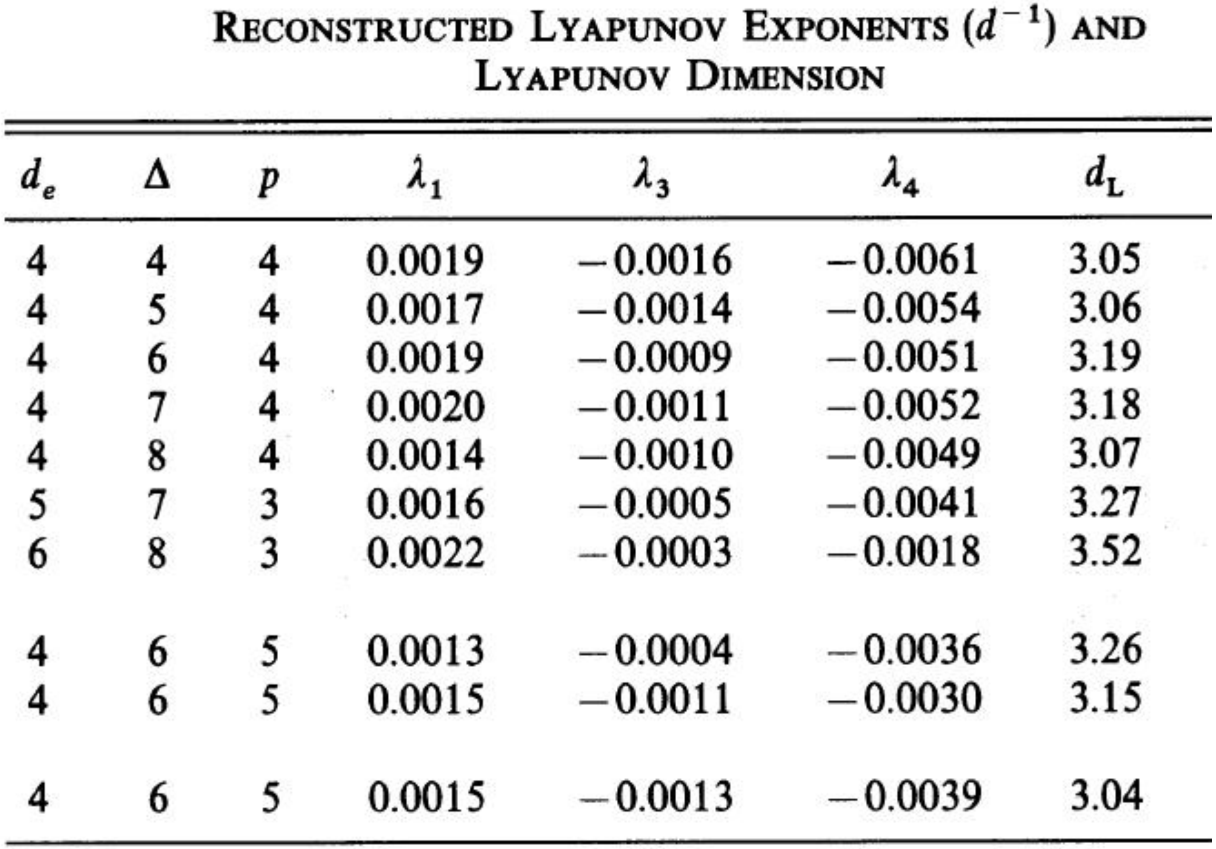
Nonlinear time-series analysis, basic steps

- smoothing and noise filtering the data
- estimating the main parameters of the phase-space
- attractor reconstruction with time-delay embedding
- calculating the Lyapunov-spectrum
- comparison with known chaotic systems

Projections in the reconstructed phase space



R Sct chaos parameters:



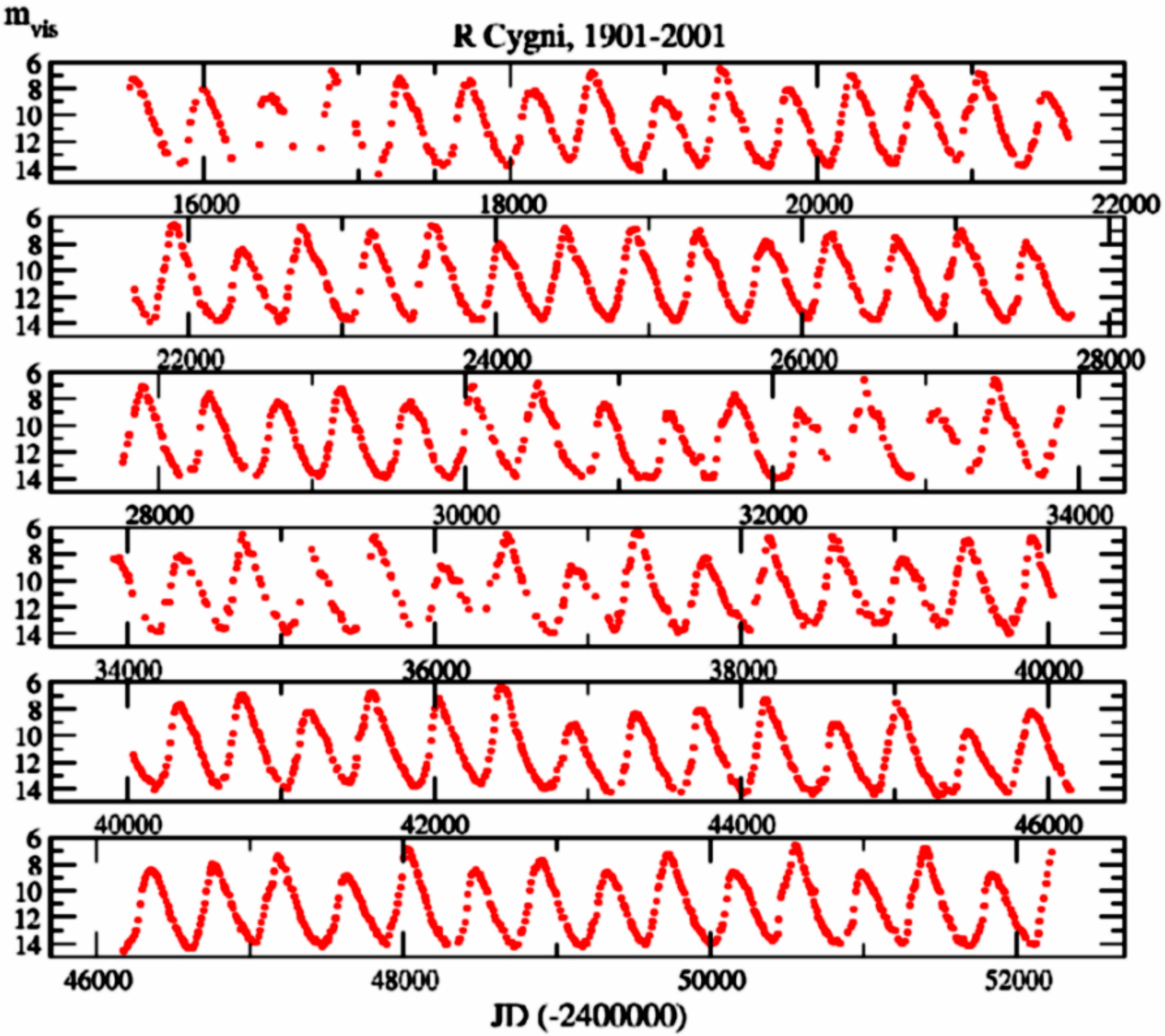
RECONSTRUCTED LYAPUNOV EXPONENTS (d^{-1}) AND LYAPUNOV DIMENSION

d_e	Δ	p	λ_1	λ_3	λ_4	d_L
4	4	4	0.0019	−0.0016	−0.0061	3.05
4	5	4	0.0017	−0.0014	−0.0054	3.06
4	6	4	0.0019	−0.0009	−0.0051	3.19
4	7	4	0.0020	−0.0011	−0.0052	3.18
4	8	4	0.0014	−0.0010	−0.0049	3.07
5	7	3	0.0016	−0.0005	−0.0041	3.27
6	8	3	0.0022	−0.0003	−0.0018	3.52
4	6	5	0.0013	−0.0004	−0.0036	3.26
4	6	5	0.0015	−0.0011	−0.0030	3.15
4	6	5	0.0015	−0.0013	−0.0039	3.04

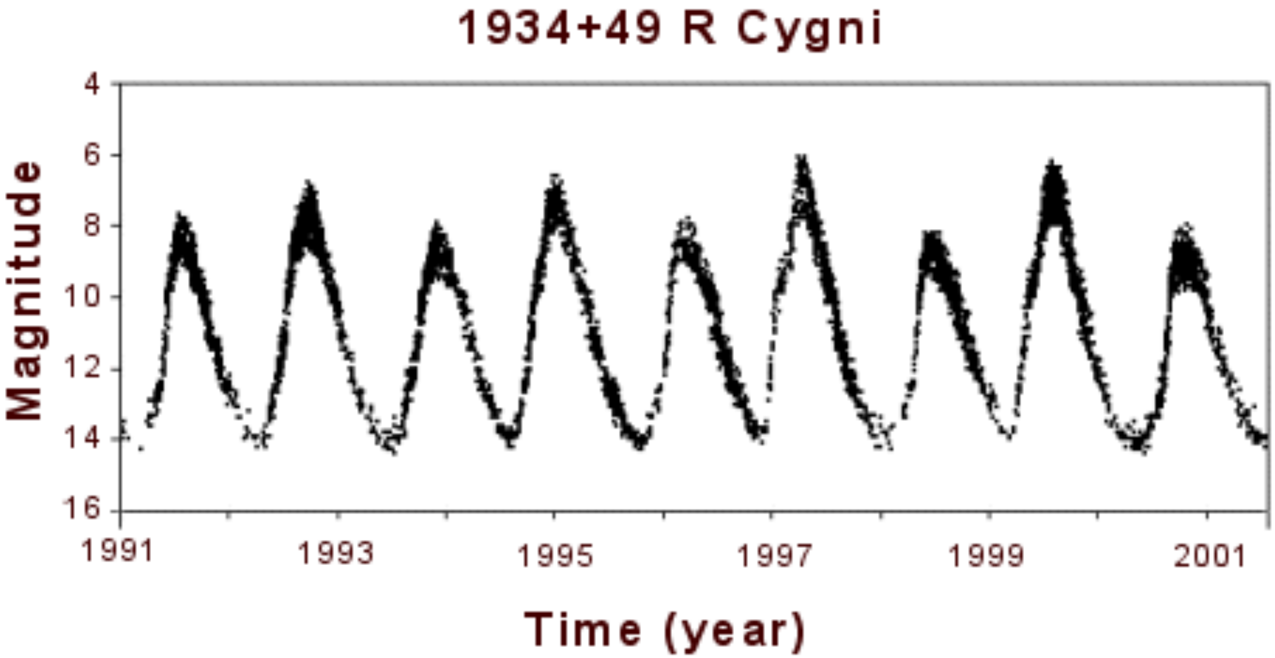
Physical conclusion: the erratic pulsational behaviour is the result of the nonlinear interaction of two mechanical modes of oscillation.

R Cygni: a Mira star pulsating chaotically

(Kiss & Szatmáry 2002)

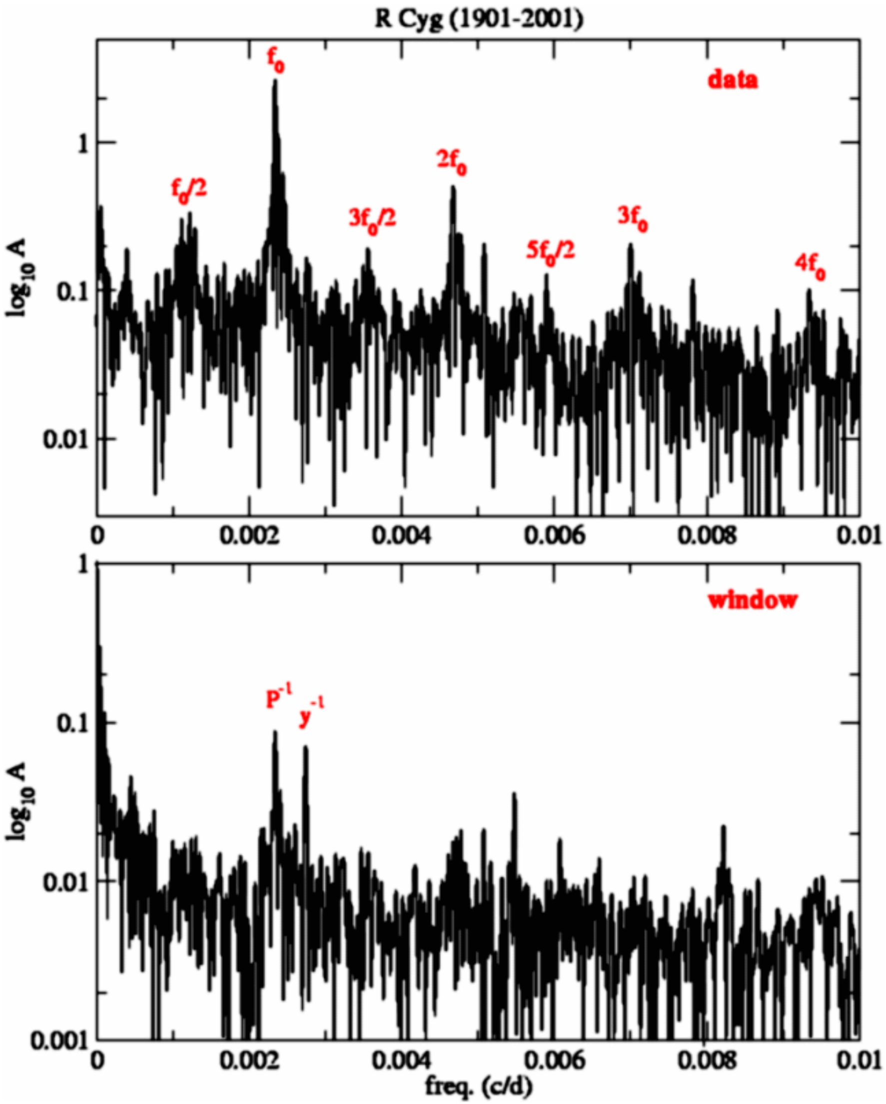


Alternating maxima: what is the true period?

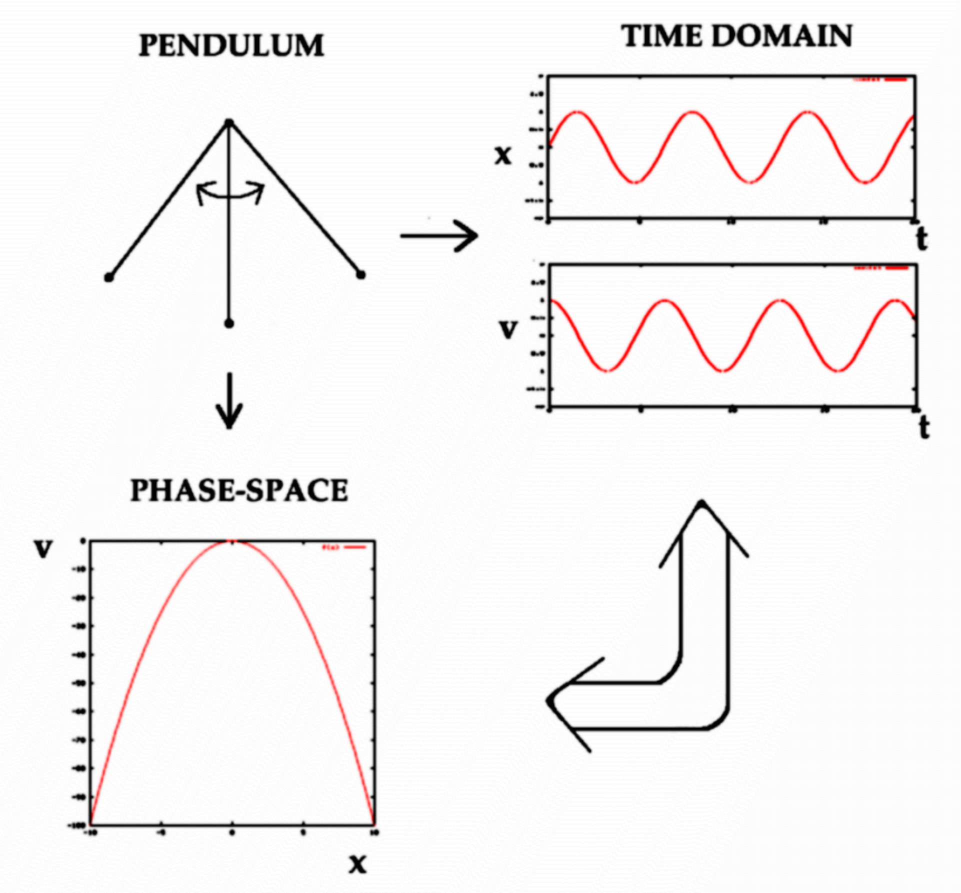


(http://www.aavso.org)

R Cyg: the Fourier-spectrum shows subharmonic components

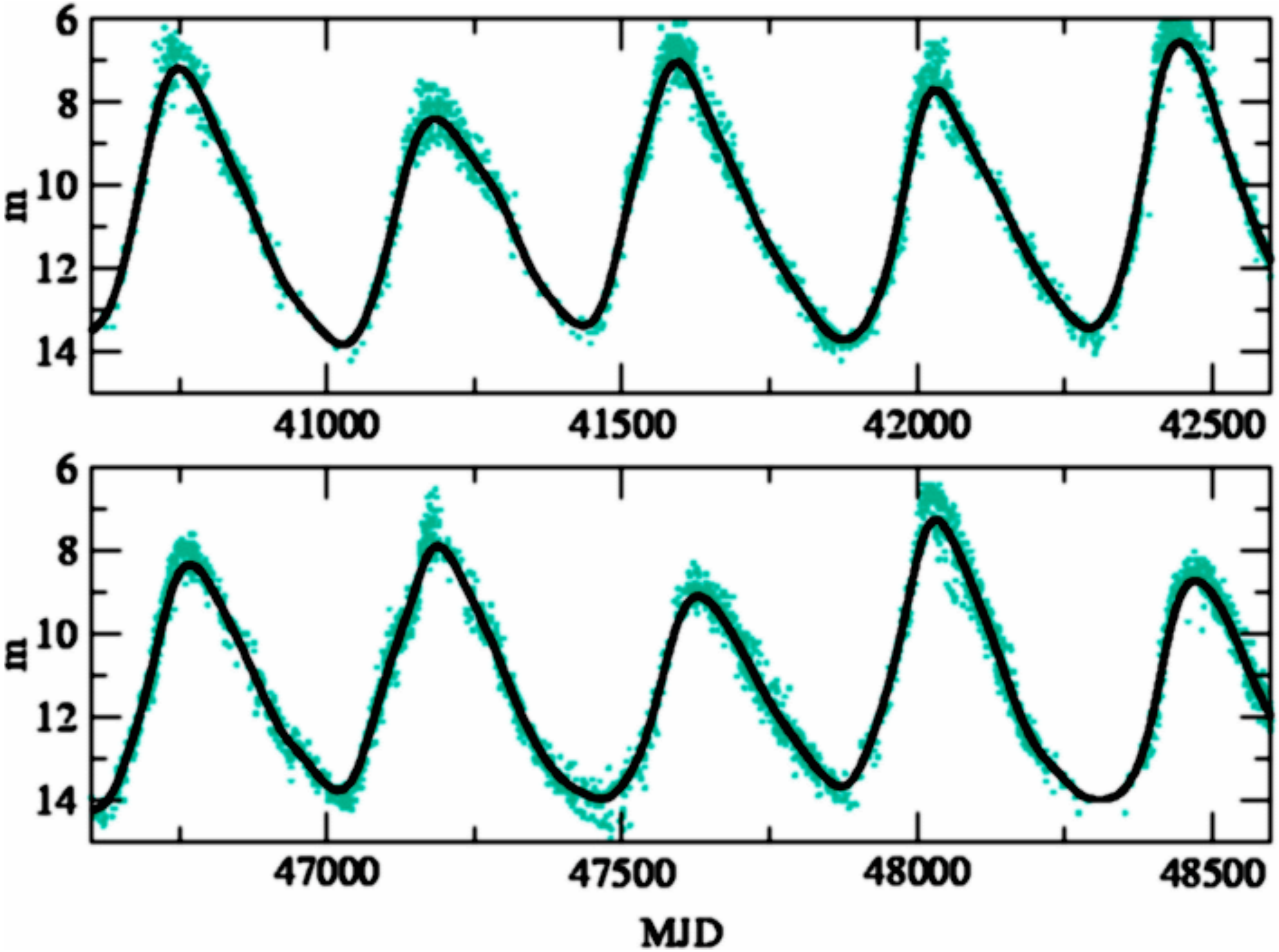


R Cygni: frequency and time-frequency analysis do not help understand the dynamics.

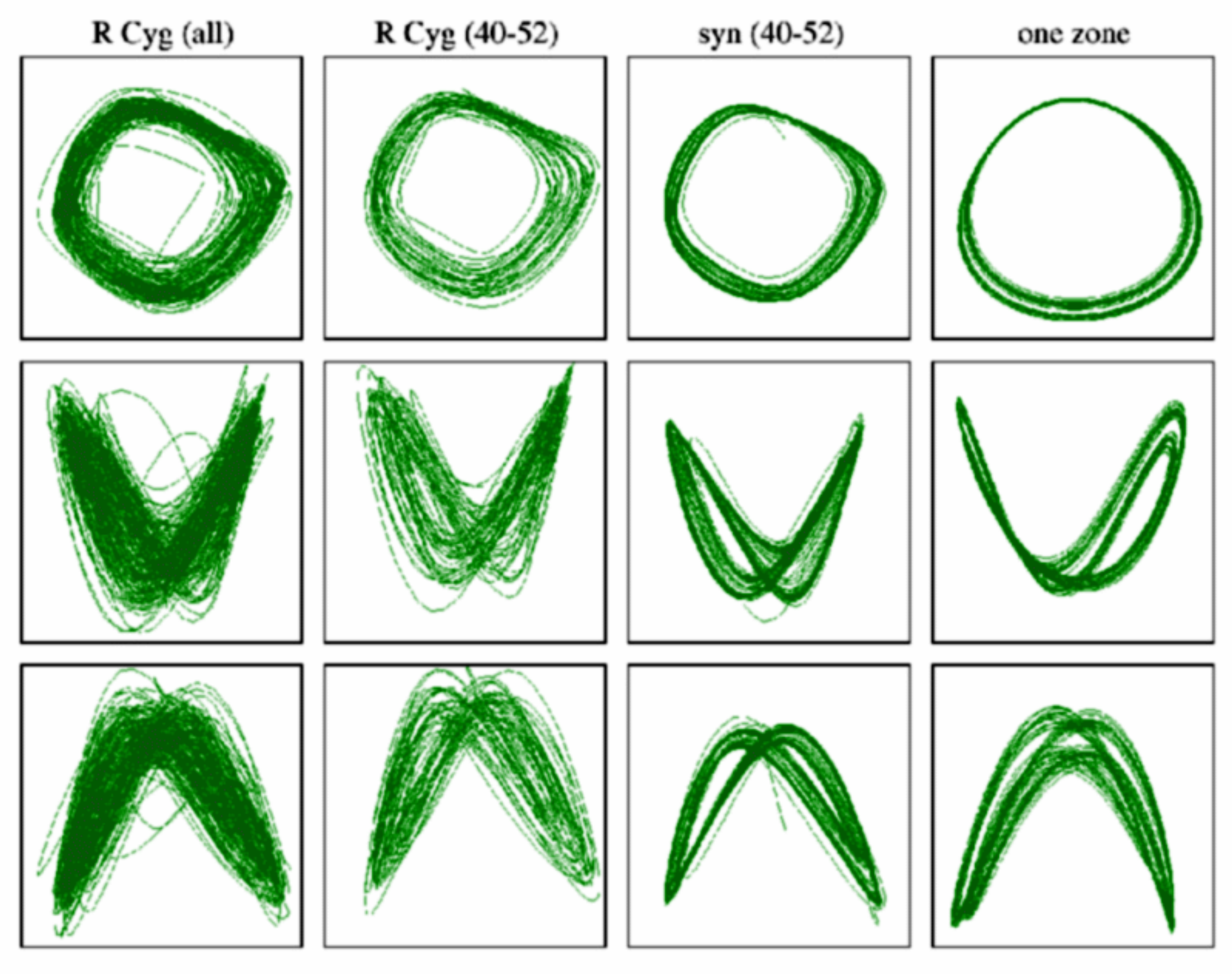


a different approach: nonlinear time-series analysis

Noise-filtered data:



Projections of the 4-D reconstructed attractors



Striking regularity and similarity to a known chaotic system ("one zone")!

R Cygni - a mathematical summary

- the phase portraits show remarkably regular structures; the optimal embedding dimension is 4
- the largest Lyapunov-exponent is positive, i.e. the system is clearly chaotic
- it shows signatures of period doubling bifurcation, so that the pulsations occur close to a period 2T orbit in the phase space
- there is a striking similarity to a known chaotic one-zone model:

$$x''' + Kx'' + x' + K\mu x(1 + \beta x) = 0 \qquad (1)$$

R Cygni: astrophysical implications

- it was the first Mira star discovered to pulsate chaotically (few has been inspected before with ambiguous conclusions)
- despite the **very complex** structure of an red giant star, its oscillations occur in a **simple** phase space of an estimated dimension 4
- the seemingly irregular light curve changes are likely to be caused by nonlinear interaction of **two** vibrationally normal modes

Period doubling in pulsating stars before Kepler

RV Tauri stars:
Definition: alternating deep and shallow minima

Period doubling in a Mira star (R Cyg)
Kiss & Szathmáry 2002

Period doubling in Cepheid, W Vir, RV Tauri models
Buchler & Kovács (1987, 1988)
Moskalik & Buchler (1991, 1992)

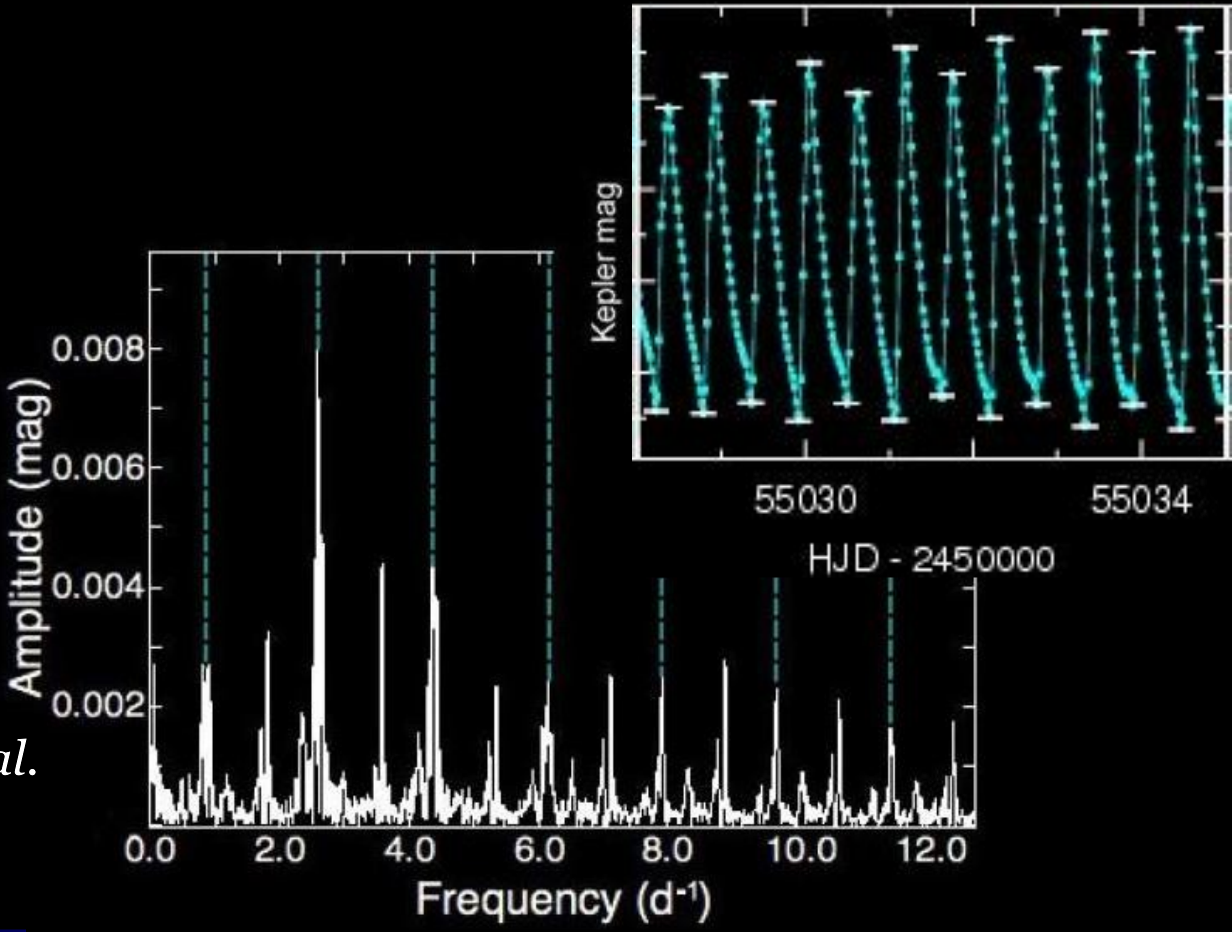
Cause of PD in the hydro models:
low order resonance between the *fundamental mode* and
a *low-order radial overtone*

$(2n+1)\,\omega_0 \approx 2\omega_k$ n=1,2 0: fundamental mode k: k th overtone

An unexpected Kepler-discovery: period doubling (PD) in RR Lyrae stars

Manifestation:

- alternating cycles
- half-integer frequencies ($1/2\ f_0$, $3/2\ f_0$, $5/2\ f_0$...)



RR Lyr Q1 Kolenberg et al. 2010

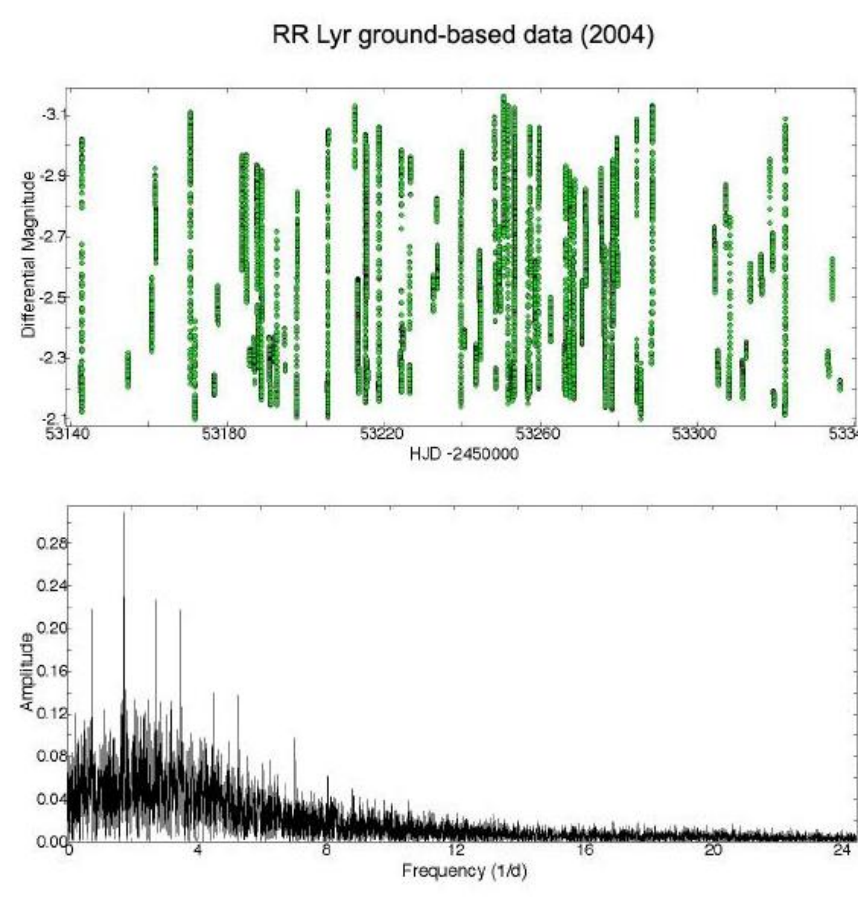
Kolenberg, Szabó, Kurtz, et al. 2010, ApJL 713, 198

Szabó, Kolláth, Molnár et al. 2010, MNRAS 409, 1244

PD has never been observed in RR Lyr stars nor in RR Lyr models.

Kepler: continuous monitoring

Kolenberg, Bryson, Szabó et al. MNRAS, 411, 1167, 2011

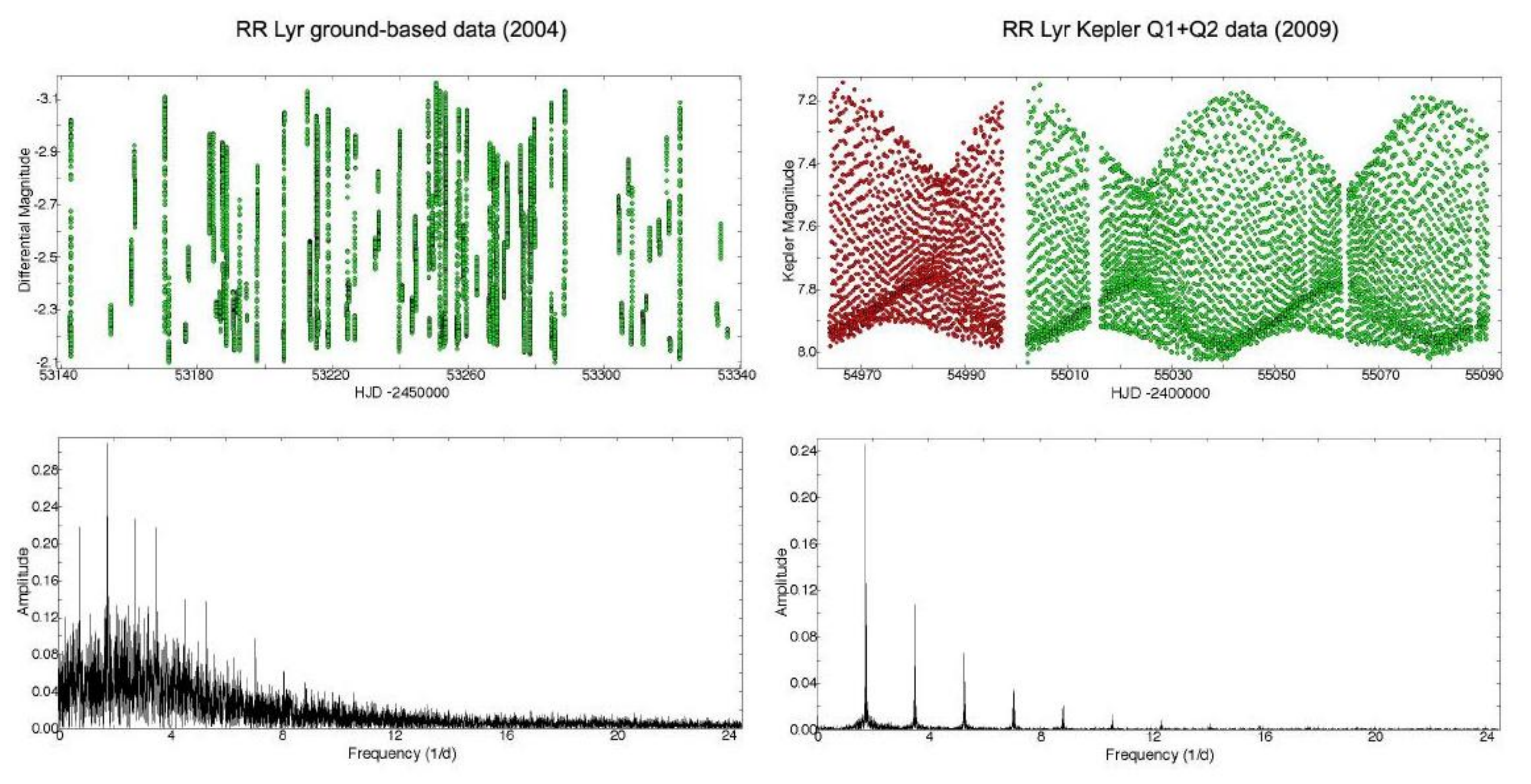


RR Lyr ground-based data (2004)

Ground-based multisite photometric campaign (6 observatories)

Kepler: continuous monitoring

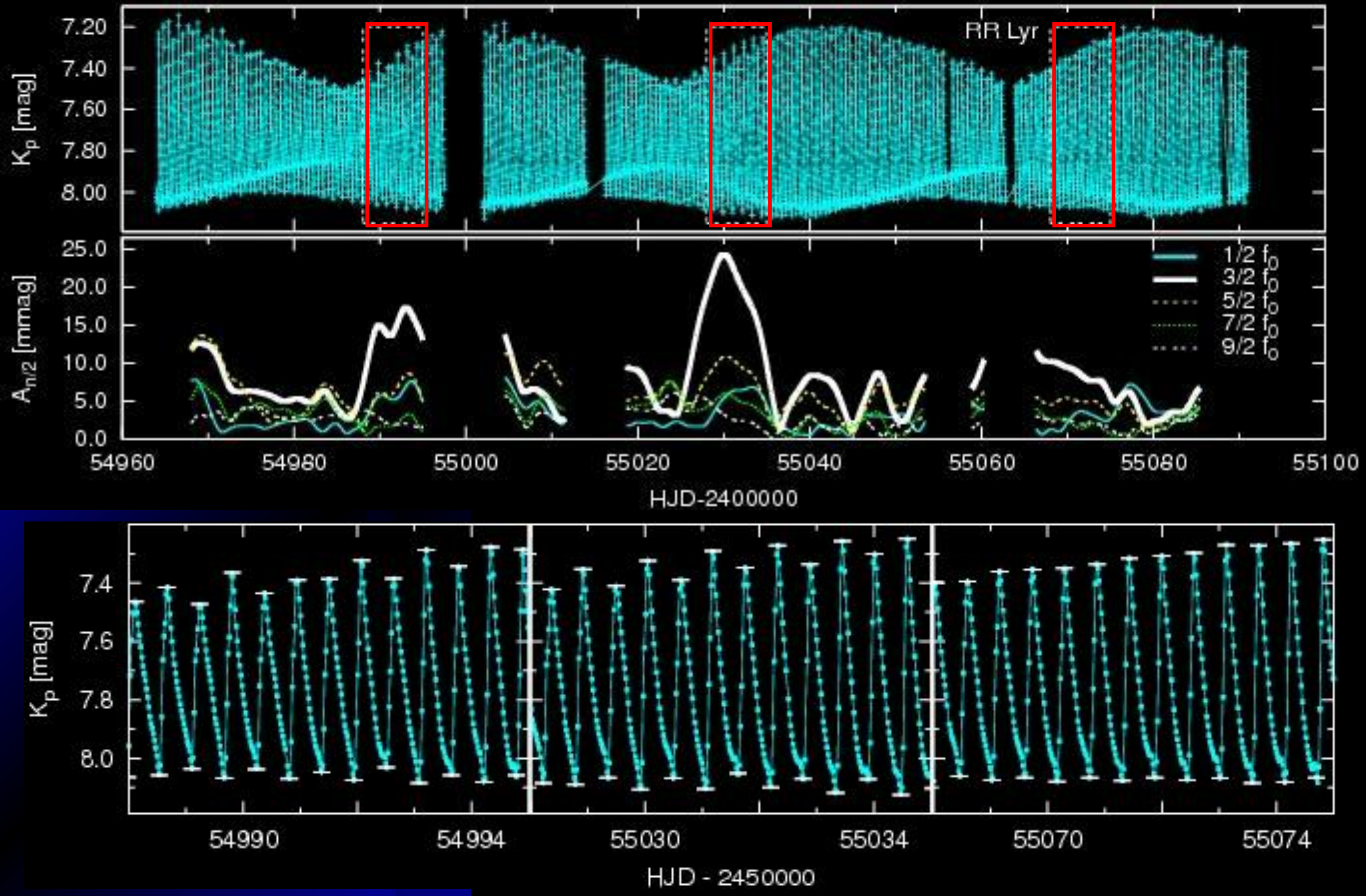
Kolenberg, Bryson, Szabó et al. MNRAS, 411, 1167, 2011



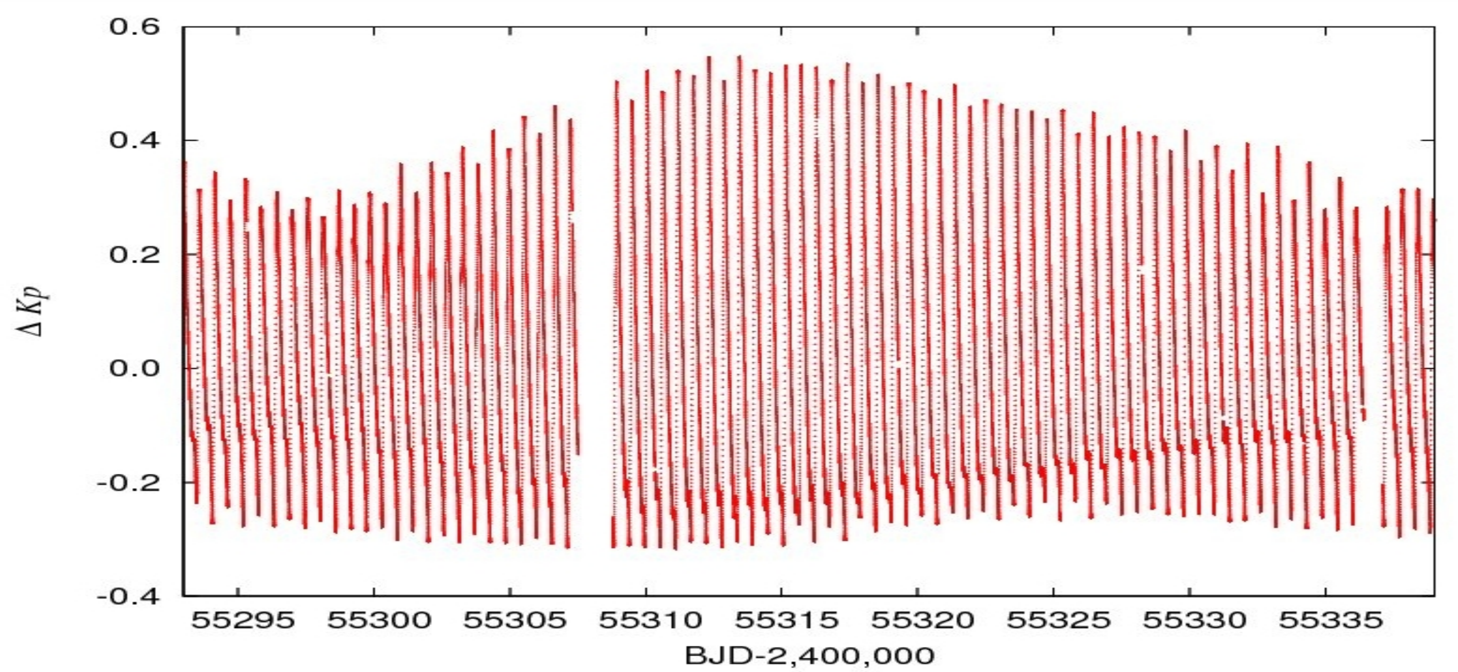
Ground-based multisite photometric campaign (6 observatories)

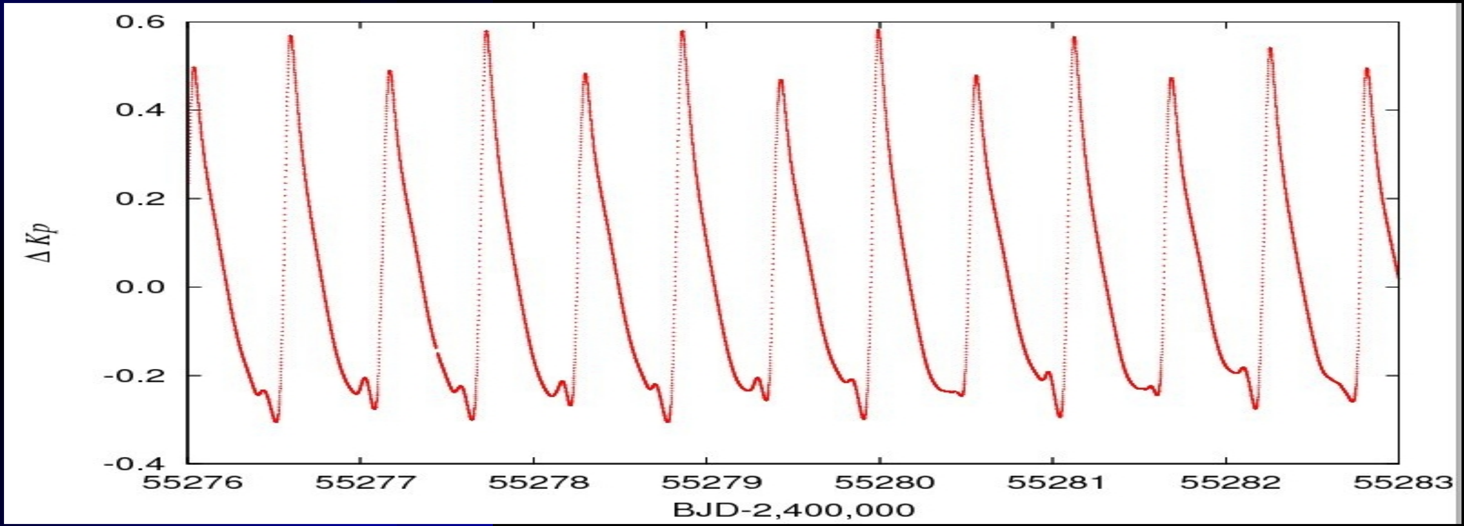
Kepler

Period doubling in RR Lyr (Q1+Q2)



A *Kepler* RR Lyr short-cadence data set





Period doubling

Period doubling:
- interesting nonlinear dynamical phenomenon

- Key to the Blazhko enigma:
  - period doubling is seen only in Blazhko stars
  - period doubling is seen in most of the Blazhko stars

Models and explanation

Hydrodynamic calculations proved that **the cause of the period doubling effect is a high order resonance (9:2) between the fundamental mode and the 9th radial overtone** (strange mode).

Szabó, R., Kolláth, Z., Molnár, L. et al. *2010, MNRAS 409, 1244*
Kolláth, Z. Molnár, L., Szabó, R. *2011, MNRAS 414, 1111*

Period doubling spin-off

PD led to the discovery of a plethora of other dynamical phenomena:

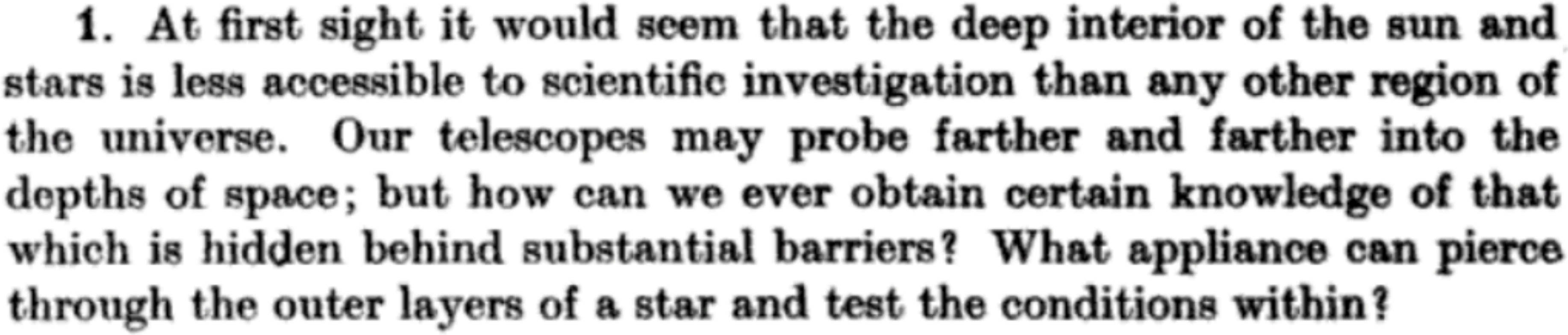
- high-order resonances (9:2)
  Szabó, Kolláth, Molnár et al. 2010, MNRAS 409, 1244

- presence of high radial overtones (strange modes)
  Kolláth, Molnár, Szabó 2011, MNRAS, 414, 1111

- presence of other radial modes (1st and 2nd overtones)
  in Blazhko stars
  Molnár, Kolláth, Szabó et al. ApJL, 2012, 757, 13

- even low-dimensional chaos
  Plachy, Molnár, Kolláth 2013, MNRAS 433, 3590

- *new explanation of the Blazhko effect*
  Buchler & Kolláth ApJ 2011, 731, 24



1. At first sight it would seem that the deep interior of the sun and stars is less accessible to scientific investigation than any other region of the universe. Our telescopes may probe farther and farther into the depths of space; but how can we ever obtain certain knowledge of that which is hidden behind substantial barriers? What appliance can pierce through the outer layers of a star and test the conditions within?

Eddington (1926): “The Internal Constitution of the Stars”