Will I get my money back?

Modeling counterparty default

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- The loss suffered by the lender is a function of the default's distribution in time, the exposure, and a recovery rate
- We can hedge our risk, mitigate it (central clearing, collateralization, etc.), and/or charge our counterparty for being credit risky (valuation adjustments)

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- "Default only happens once at a time"
- This resembles a Poisson-process

Poisson-process (homogenous)
Let $\lambda > 0$ be fixed. The process $\{N(t),\, t \in [0, \infty)\}$ is a Poisson-process with rate/intensity λ if all of the following conditions hold:

1. $N(0) = 0$
2. N has independent and stationary increments
3. The following are true:
    1. $\mathbb{P}(N(h) = 0) = 1 - \lambda h + o(h)$
    2. $\mathbb{P}(N(h) = 1) = \lambda h + o(h)$
    3. $\mathbb{P}(N(h) \geq 2) = o(h)$

The default

Poisson process

As a result, for a homogenous Poisson-process the number of "arrivals" in any interval of length $h > 0$ has a $Poisson(\lambda h)$, distribution, that is,

$$\mathbb{P}(N(h) = 0) = e^{-\lambda h}$$

In our practical case we'll consider an inhomogenous Poisson-process, that is, the intensity will be time-dependent. For an interval (t_1, t_2)

$$P(N(t_1, t_2) = 0) = e^{-\int_{t_1}^{t_2} \lambda(t) \mathrm{d}t}$$

Interest rates and discounting

Given the instantaneous short rate $r(t)$, the price of a zero-coupon bond at time t for expiry $T \geq t$ is

$$D(t,T) = \mathbb{E}^{\mathbb{P}}\left[e^{-\int_t^T r(s)\mathrm{d}s} \,\middle|\, \mathcal{F}_t\right]$$

We can define instantaneous forward rate f as

$$\begin{aligned} D(t,T) &= e^{-\int_t^T f(t,s)\mathrm{d}s} \\ f(t,s) &= -\frac{\partial \ln D(t,s)}{\partial s} = -\frac{1}{D(t,s)} \cdot \frac{\partial D(t,s)}{\partial s} \end{aligned}$$

and continuously compounded yield or zero rate as

$$\mathrm{R}(t,T) = -\frac{1}{T-t} \ln D(t,T)$$

Hazard rates

Default is assumed to be an **inhomogenous Poisson-process** with a hazard rate $\lambda(t)$ at time t. Then for the default time τ we introduce the following notation $S(t,T)$ for $T > t$:

$$\mathbb{P}(t < \tau \leq t + \mathrm{d}t \,|\, \tau > t) = \lambda(t)\mathrm{d}t$$

$$S(t,T) = \mathbb{P}(\tau > T \,|\, \tau > t) = \mathbb{E}(\chi_{\tau > T} \,|\, \mathcal{F}_t) = \mathbb{E}^{\mathbb{P}}\left[e^{-\int_t^T \lambda(s)\mathrm{d}s} \,|\, \mathcal{F}_t\right]$$

In case λ is deterministic, $Q(t,T) = e^{-\int_t^T \lambda(s)\mathrm{d}s}$.
Analogously the **forward hazard rate** h can be defined as

$$S(t,T) = e^{-\int_t^T h(t,s)\mathrm{d}s}$$
$$h(t,s) = -\frac{\partial \ln S(t,s)}{\partial s} = -\frac{1}{S(t,s)} \cdot \frac{\partial S(t,s)}{\partial s}$$

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- Protection seller provides protection on default : In the event of the default of the reference entity the protection seller pays the buyer the agreed notional (see examples below)
- The price of this protection (i.e., the premium paid) reflects the market's opininon on the credit riskyness of the reference entity

Credit Default Swap (CDS)

Physical settlement

In the original setup, periodic payments are the implied fair price of the protection. In case of default the protection buyer and seller exchange the defaulted asset for par.

Protection buyer:

- periodic premium payments
- in case of default delivers a default reference asset

Protection seller

- provides protection of default:
  - in case of default pays par/notional

Credit Default Swap (CDS)

Cash settlement

In case a defaulted asset is not available, an auction is conducted to establish the recovery rate of the asset, and the contract is cash settled.

Protection buyer:

- periodic premium payments
- ~~in case of default delivers a default reference asset~~

Protection seller

- provides protection of default:
  - in case of default pays *par - recovery value*

Credit Default Swap (CDS)

Cash settlement standardized

In April 2009 CDS contracts became standardized in many ways. Coupons are regularized now, ending up having a mismatch in ther two legs of the trade. Entry prices/upfront fees are introduced due to this.
Accrual period end dates and maturities are fixed standard dates based on start dates.

Protection buyer:

- *pays/receives upfront fee*
- *quarterly standard* premium payments

Protection seller

- *pays/receives upfront fee*
- provides protection of default:
  - in case of default pays par - recovery value

- For sake of simplicity we will assume that the valuation days is day 0
- The i-th accrual period has start date s_i, end date e_i and payment date p_i
- $DCC(t_1, t_2)$ is the day count fraction between dates t_1 and t_2
- M is the maturity date of the trade
- N is the notional of the contract
- C is the coupon/premium quoted in bps of the notional, annualized
- $R(\tau)$ is the recovery rate at τ
- $h(t) = h(0, t)$, $S(t) = S(0, t)$

CDS Pricing

Protection leg

- Random payment of $N(1-R(\tau))$ at τ

$$PV_{ProtectionLeg} = N \cdot \mathbb{E}\left[e^{-\int_0^\tau r(s)\mathrm{d}s} \cdot (1-R(\tau)) \cdot \chi_{\tau \leq M}\right]$$

- Assuming that the recovery rate is independent of the interest rate, hazard rate and τ

$$PV_{ProtectionLeg} = N \cdot (1-R) \cdot \mathbb{E}\left[e^{-\int_0^\tau r(s)\mathrm{d}s} \cdot \chi_{\tau \leq M}\right]$$

- Assuming that interest rates and hazard rates are independent

$$PV_{ProtectionLeg} = -N \cdot (1-R) \cdot \int_0^M D(t)\mathrm{d}S(t) =$$

$$= N \cdot (1-R) \cdot \int_0^M D(t)h(t)S(t)\mathrm{d}t$$

CDS Pricing

Premium leg - Standard coupon payments

- In the i-th quarter the protection buyer pays $N \cdot DCC(s_i, e_i) \cdot C$ on date p_i, pays an accrued $N \cdot DCC(s_i, \tau) \cdot C$ at default
- Let U be the number of remaining payments, then

$$PV_{PremiumsOnly} = N \cdot C \cdot \mathbb{E}\left[\sum_{i=1}^{U} DCC(s_i, e_i) \cdot D(p_i) \cdot \chi_{e_i \leq \tau}\right] =$$

$$= N \cdot C \cdot \sum_{i=1}^{U} DCC(s_i, e_i) \cdot D(p_i) \cdot S(e_i)$$

- In case of default the protection buyer pays the premium accrued until default

$$PV_{Accrued} = N \cdot C \cdot \mathbb{E}\left[\sum_{i=1}^{U} DCC(s_i, e_i) \cdot e^{\int_0^{\tau} r(s)\mathrm{d}s} \cdot \chi_{s_i \le \tau \le e_i}\right] =$$

$$= -N \cdot C \cdot \sum_{i=1}^{U} \int_{s_i}^{e_i} DCC(s_i, t) D(t) \mathrm{d}S(t)$$

- The value of the premium leg is the sum of the coupons and accruals.

$$PV_{PremiumLeg} = PV_{PremiumsOnly} + PV_{Accrued} = C \cdot RPV01_{dirty}$$

where $RPV01$ is the so-called dirty risky PV01 of the trade.

CDS Pricing

- The dirty price of the CDS can be then calculated as

$$PV_{dirty} = PV_{ProtectionLeg} - C \cdot RPV01_{dirty}$$

- The dirty price has a sawtooth pattern against time. To smooth it out, the accrued interest between the accrued start date and the step-in date is subtracted,

$$\begin{aligned} RPV01_{clean} &= RPV01_{dirty} - N \cdot DCC(s_1, t_e) \\ PV_{clean} &= PV_{ProtectionLeg} - C \cdot RPV01_{clean} \end{aligned}$$

CDS Pricing

Par Spread

Prior to the Big Bang in 2009, CDS contract were entered into at 0 price, paying fair premia for the protection. The coupon that makes the clean PV of the CDS 0 is known as the *par spread*:

$$C_P = \frac{PV_{ProtectionLeg}}{RPV01_{clean}}$$

- A usual approximation of the PV is to assume that all cash flows related to the default happen at accrual period end dates, that is,

$$PV_{ProtectionLeg} \approx N(1-R)\sum_{i=1}^{U} D(t_i)\cdot(S(t_{i-1})-S(t_i))$$

- If in addition we assume that on average default happens in the middle of periods,

$$PV_{Accrued} \approx 0.5\cdot NC\sum_{i=1}^{U} DCC(s_i,e_i)D(t_i)\left(S(t_{i-1})-S(t_i)\right)$$

and so

$$RPV01_{dirty} \approx N\cdot\sum_{i=1}^{U} DCC(s_i,e_i)D(t_i)\left[\frac{S(t_{i-1})+S(t_i)}{2}\right]$$

CDS Pricing

Common approximations II

- If the premium leg were paid continuously, there would be no accrual, and so

$$PV_{Premium,Continuous} = N \cdot C \cdot \int_0^M D(t)S(t)\mathrm{d}t$$

and so the par spread would be given as

$$C_P = -(1-R)\frac{\int_0^M D(t)\mathrm{d}S(t)}{\int_0^M D(t)S(t)\mathrm{d}t}$$

- If in addition we consider a constant hazard rate, above expression can be reduced to the so-called credit triangle relationship :

$$C_P = (1-R)\cdot\lambda\cdot\frac{\int_0^M D(t)e^{-\lambda t}\mathrm{d}t}{\int_0^M D(t)e^{-\lambda t}\mathrm{d}t} = (1-R)\lambda$$

Practical usage and issues

- Credit Valuation Adjustment (CVA) is charged to a counterparty for being credit risky, that is, on a very high level equals

$$CVA = \int_0^M D(t) \cdot Exposure(t) \cdot R(t) \mathrm{d}S(t)$$

  that is, for the caculations we need to know survival probabilities.
- In theory, if we knew hazard rates, we could price the CDS fairly. In practice this happens backwards : observe market prices/premia of CDS-s, and calibrate hazard rate models accordingly.
  - CDS quotes exist only at some standard traded maturities - a handful of points, from which we want to recreate an entire curve
  - Hazard curve models; numerical approaches
- In case of counterparties that have no liquid CDS on the market, we still have to come up with a model
  - Regression approaches or manual curve selections based on similarities
  - Population issues, considering liquidities.

Q/A