Income Distibution and Tax Policies

# Individual Income

We model the production of an individual as:

$$x=azε$$

where $a>0$ is her ability, $z>0$ is her effort, and $ε>0$ is a luck factor, a random variable with unit mean and unit variance. The individual-specific luck factors are iid. We assume that the gross income of the individual is equal to her production.

## Model 1

First we assume zero tax, therefore the individual’s net income equals to her gross income, that is, to her production.

$$I=x$$

We model the utility of the individual as:

$$U=E\left[I\right]-\frac{c}{2}z^{2}$$

where $c>0$ is an individual factor determining the disutility of effort. We can associate $1/c$ with diligence. Let us plug in our model to the utility expression. We obtain:

$$U=E\left[azε\right]-\frac{c}{2}z^{2}=azE\left[ε\right]-\frac{c}{2}z^{2}$$

$$U=az-\frac{c}{2}z^{2}$$

Every individual is aware of her attributes, and chooses her effort to maximize her utility. We obtain the optimal effort as:

$$\hat{z}=\frac{a}{c}$$

This shows that more diligent individuals, and individuals with more ability work harder.

Plugging this result into the income expression we obtain the optimized income:

$$\hat{I}=\hat{x}=\frac{a^{2}ε}{c}$$

This shows that more diligent individuals, and especially individuals with more ability obtain higher income. Luck of course remains a factor.

## Model 2

We still assume zero tax, but now we take into account the risk-aversion of the individual. We use a mean-variance type utility:

$$U=E\left[I\right]-\frac{γ}{2}Var\left[I\right]-\frac{c}{2}z^{2}$$

where $γ>0$ is the individual’s risk-awareness parameter. Plugging in, we obtain:

$$U\left(z\right)=az-\frac{γa^{2}}{2}z^{2}-\frac{c}{2}z^{2}$$

Again, we obtain the optimal effort optimizing with respect to $z$:

$$\hat{z}=\frac{a}{c+γa^{2}}$$

This shows that more diligent individuals will work harder, but the dependence on ability is not monotonous!

Plugging this result into the income expression we obtain the optimized income:

$$\hat{I}=\frac{a^{2}ε}{c+γa^{2}}$$

This shows that more diligent individuals, and risk-takers obtain higher income. Dependence on ability is monotonous, but it saturates. Luck of course remains a factor.

A proportional income tax with tax rate $t$ is subtracted. So her net income from her work, after tax is $x\left(1-t\right)$ and the tax she pays is $xt$. The collected tax is redistributed among the individuals uniformly. We can thus write the net income of the individual as

$$I=x\left(1-t\right)+\left〈x\right〉t=x+\left(\left〈x\right〉-x\right)t$$

where $\left〈x\right〉$ is the population-average of the individual productions. The population average net income is of course equal to the population average gross income: $\left〈I\right〉=\left〈x\right〉$ as can be easily checked.

We write the utility of the individual as:

$$U=E\left[I\right]-\frac{γ}{2}Var\left[I\right]-\frac{c}{2}z^{2}$$

where $γ>0$ is her risk-aversion, and $c>0$ is her effort-disutility parameter. The expectation and variance are defined over the random $ε$.

We consider a very large population (mathematically a continuum of individuals with unit weight), with some joint distribution of the individual-specific attributes $a$, $γ$ and $c$.

Every individual is aware of her attributes, and chooses her effort to maximize her utility.

$$U\left(z\right)=\left(1-t\right)az-\frac{γ\left(1-t\right)^{2}a^{2}}{2}z^{2}-\frac{c}{2}z^{2}+\left〈x\right〉t$$

Note that as the population is large, the population average completely averages out the idiosynchratic random factors, the redistribution term has zero variance. Also note that the individual effort has negligible (zero) effect on the redistributed benefit, so it does not factor into the optimization. Optimizing the utility we obtain the optimal effort as:

$$\hat{z}=\frac{\left(1-t\right)a}{c+γ\left(1-t\right)^{2}a^{2}}$$

At this optimal effort, gross income becomes:

$$\hat{x}=\frac{\left(1-t\right)a^{2}ε}{c+γ\left(1-t\right)^{2}a^{2}}$$

Averaging out the luck factor we obtain:

$$E\left[\hat{x}\right]=\frac{\left(1-t\right)a^{2}}{c+γ\left(1-t\right)^{2}a^{2}}$$

Net income after averaging out the luck factor:

$$E\left[\hat{I}\right]=\frac{\left(1-t\right)^{2}a^{2}}{c+γ\left(1-t\right)^{2}a^{2}}+t\left〈\frac{\left(1-t\right)a^{2}}{c+γ\left(1-t\right)^{2}a^{2}}\right〉$$