Road surveillance optimization - an asymmetric vehicle routing problem with visiting frequencies

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Overview

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Background

- In Hungary, public road are maintained by the Hungarian Public Road Nonprot Private Limited Company
- They use special surveillance cars to regularly check the road network.
  - Each road must be checked in a law regulated frequency, depending its type, traffic etc.
- This is currently organized by hand (using a GIS system + spreadsheets)
- They want something better, thus initiated a pilot project to
  - evaluate the current situation and the effect of some changes in the organization
  - automatize the planning process at a later state

Goals

Task

The pilot project deals with a specific county (Bács-Kiskun) of Hungary.

- 6 cars with fixed centers/garages (two of them are co-located)
  - Each road is (currently) assigned to one of the centers.
- different speed when surveying or just traveling (given per road)
- daily full travel time is limited: max 9 hours
- visiting frequency depends on road type: 1, 2, 7, 14, 28 days
  - We need a periodic plan for 28 days

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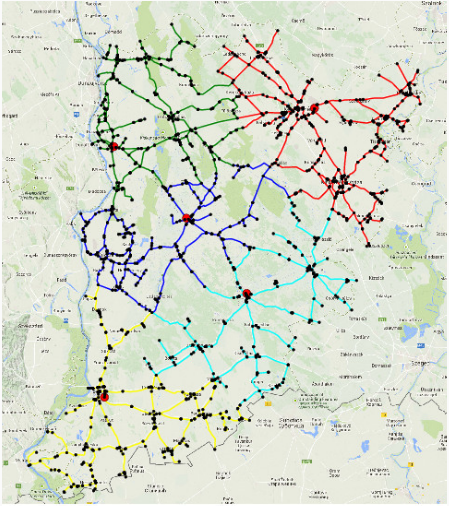
Questions: Optimize the daily routes

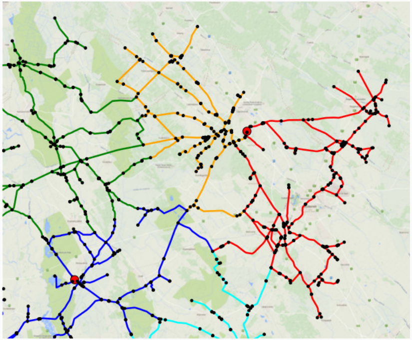
1. while keeping all the centers and the current road assignment ("borders")
2. with optimized road $\rightarrow$ center assignment
    - the assignment may be static or dynamic
3. with using less cars

Overview

Input data

- 8 files (.dbf, .shp, .shx, .txt formats)
- 1564 nodes, 3241 road segments
  - road length, speed limits, one-way/bidirectional, visiting frequency





Overview

Math Formulation

Given

$freq : E \longrightarrow \mathbb{N}$**.** The maximum (or exact) number of days between two consecutive checks of the road segment.

$time_{trv} : E \longrightarrow \mathbb{R}$ **and** $time_{srv} : E \longrightarrow \mathbb{R}$**.** The traveling and surveying time of the road segments

$length : E \longrightarrow \mathbb{R}$**.** The physical length of the road segments.

$bidir : E \longrightarrow \{0, 1\}$**.** One-way/bidirectional indicator.

$centers : E \longrightarrow 2^{\mathcal{C}}$**.** The set of vehicles (i.e. centers) which are allowed to survey the road segment.

$\mathcal{T} := 0, \ldots, T-1$**.** The planning time frame.

W**.** Daily max working time.

Math Formulation

Find

$p_{c,d}$ (closed) paths and a set $S_{c,d} \subset p_{c,d}$ of surveyed roads ($\forall c \in \mathcal{C}$ and $\forall d \in \mathcal{T}$) so that

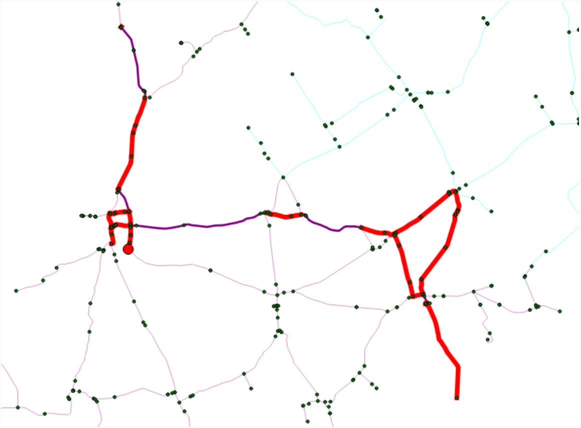
- $p_{c,d}$ starts from and ends at c,
- $S_{c,d} \subset \{e : c \in centers(e)\}$, i.e. each road is surveyed by one of its allowed vehicles,
- $\sum_{e \in S_{c,d}} time_{srv}(e) + \sum_{e \in p_{c,d} \setminus S_{c,d}} time_{trv}(e) \leq W$, i.e. the total travel time of $p_{c,d}$ is at most W,
- $e \in \bigcup \{S_{c,k} : c \in \mathcal{C}, k \in \{d, (d+1)_{modT}, \ldots, (d + freq(e) - 1)_{modT}\}$ holds for each $e \in E$, $freq(e) > 0$, and $d \in [0, \ldots, T-1]$.

Objective function

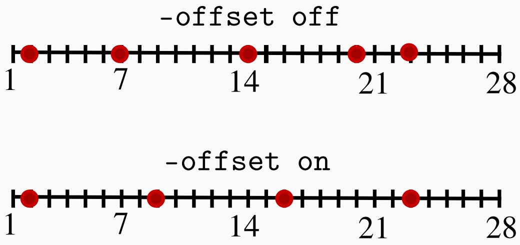
we seek to minimize the total length of all paths $p_{c,d}$, i.e

$$\sum_{c \in \mathcal{C}, d \in [0, \ldots, R-1]} length(p_{c,d}). \qquad (1)$$

An example single day route plan



Exact or min. visiting frequency (day patterns)



Remark

According to our tests, it doesn't have much effect on the found solution.

Overview

Insertion Heuristic

Algorithm

1. Start with empty routes p_{cd} for each center c and day d
2. Choose and arbitrary unchecked road e,
3. Calculate the cost of its optimal insertion into each route routes p_{cd}.
4. Choose the best center and survey day pattern
5. GOTO Step 2 if there are unchecked roads.

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Solving Step 4

- It is done by solving a tiny set covering problem, or
- one may require that the visiting frequencies are strictly kept.

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Remarks

- It is blazing fast (solves the full problem within 1sec)
- Results are surprisingly good.
- Working day limits can be enforced by penalty functions.

Improved Heuristics

Reinsertion Algorithm

1. Create an initial solution
2. Try to remove and optimally reinsert each road until improvement is possible

Simulated Annealing [sketch]

1. Create an initial solution
2. Repeat
   - Choose a random road, and reinsert it optimally to a randomly chosen center and day offset.
   - Accept the change with probability $P(accept) := max\left\{1, e^{\frac{cost_{prev} - cost_{new}}{T}}\right\}$

Overview

MIP Formulation

$$\min \sum_{cde} lenght(e) s_{cde} + \sum_{cde} lenght(e) t_{cde}$$

where

$$s_{cde} \in \{0,1\} \qquad \forall c \in \mathcal{C}, d \in \mathcal{T}, e \in E_c$$

$$t_{cde} \in \mathbb{N} \qquad \forall c \in \mathcal{C}, d \in \mathcal{T}, e \in E_c$$

$$\sum_{k \in \{d, \ldots, (d+freq(e)-1)_{mod T}\}} s_{cke} \geq 1 \qquad \forall c \in \mathcal{C}, d \in \mathcal{T}, e \in E_c$$

$$\sum_{e \in \rho_{E_c}(v)} s_{cde} + \sum_{e \in \rho_E(v)} t_{cde} = \sum_{e \in \delta_{E_c}(v)} s_{cde} + \sum_{e \in \delta_E(v)} t_{cde} \qquad \forall c \in \mathcal{C}, d \in \mathcal{T}, v \in V$$

$$\sum_{e \in E_c} time_{srv}(e) s_{cde} + \sum_{e \in E} time_{trv}(e) t_{cde} \leq W \qquad \forall c \in \mathcal{C}, d \in \mathcal{T}$$

$$f_{cde} \geq 0 \qquad \forall c \in \mathcal{C}, d \in \mathcal{T}, e \in E_c$$

$$f_{cde} \leq M(s_{cde} + t_{cde}) \qquad \forall c \in \mathcal{C}, d \in \mathcal{T}, e \in E_c$$

$$\sum_{e \in \rho_E(v)} f_{cde} - \sum_{e \in \delta_E(v)} f_{cde} = \sum_{e \in \rho_{E_c}(v)} s_{cde} \qquad \forall c \in \mathcal{C}, d \in \mathcal{T}, v \in V - c$$

Solution I

- Too big to be solved directly by a MIP solver
- Iterative rounding may help
    1. Solve the LP relaxation
    2. Choose a variable close to 1 (or 0)
    3. Round it, then GOTO 1.
- This works but still very slow and usually doesn't give better results than the heuristics.

Solution II. Cutting Plane Approach

Alternative connectivity constraints

Exact version

$$\sum_{e \in \delta_{E_c}(U)} s_{cde} + \sum_{e \in \delta_E(U)} t_{cde} \geq \frac{1}{M} \sum_{(uv) \in E_C; u,v \in U} s_{cd(uv)} \qquad \forall c \in \mathcal{C}, d \in \mathcal{T}, U \subset V - c$$

Restricted version
Added only if unconnected integer solution is found

$$\sum_{e \in \delta_{E_c}(U)} s_{cde} + \sum_{e \in \delta_E(U)} t_{cde} \geq 1$$

Where U is the vertex set of a disconnected cycle

- The subroutine finding a violated constraint is passed to the MIP solver as a callback function.

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  However

- Assume that surveying center and the day pattern are fixed for each road.
- Then, we can find an optimal solution to this sub-problem in a reasonable time.
- So, it can be used to further improve the results calculated by the heuristics.

Overview

Results I: Insertion versus Simulated annealing

Center	Survey km/day	Travel km/day		
		Insertion	Simann	Improvement
C0	87.8	82.4	83.3	-1.1%
C1	75.8	64.5	62.6	2.9%
C2	67.7	63.8	56.8	11.0%
C3	58.7	83.2	77.6	6.7%
C4	54.3	48.0	41.3	14.0%
C5	51.4	72.1	62.6	13.2%
Total	**395.7**	**414.0**	**384.3**	**7.2%**

Results II: Insertion method and MIP-impr.

Center	Survey km/day	Travel km/day		
		Insertion	+MIP-impr.	Improvement
C0	87.8	82.4	79.3	3.8%
C1	75.8	64.5	62.2	3.6%
C2	67.7	63.8	62.0	2.8%
C3	58.7	83.2	80.6	3.1%
C4	54.3	48.0	43.7	9.0%
C5	51.4	72.1	70.1	2.8%
Total	**395.7**	**414.0**	**397.9**	**3.9%**

Results III: Simulated Annealing and MIP-impr.

Center	Survey km/day	Travel km/day		
		Simann	+MIP-impr.	Improvement
C0	87.8	83.3	82.1	1.4%
C1	75.8	62.6	60.4	3.5%
C2	67.7	56.8	55.3	2.6%
C3	58.7	77.6	76.4	1.5%
C4	54.3	41.3	39.3	4.8%
C5	51.4	62.6	61.8	1.3%
Total	**395.7**	**384.2**	**375.3**	**2.3%**

Results IV: Insertion vs. SimAnn with MIP-impr.

Center	Survey km/day	Travel km/day		
		Insertion +MIP-impr.	Simann +MIP-impr.	Improvement
C0	87.8	79.3	82.1	-3.5%
C1	75.8	62.2	60.4	2.9%
C2	67.7	62.0	55.3	10.8%
C3	58.7	80.6	76.4	5.2%
C4	54.3	43.7	39.3	10.0%
C5	51.4	70.1	61.8	11.8%
Total	**395.7**	**397.9**	**375.3**	**5.7%**

Thank you for the attention!