Matematika A1

Komplex számok

Megoldások

1.

(a) $5-i$

(b) $3+11i$

(c) $-\frac{2}{5}-\frac{29}{5}i$

(d) $\frac{4}{5}+\frac{7}{5}i$

2.

(a) $-\frac{3}{2}+\frac{\sqrt{3^3}}{2}i$

(b) $2\sqrt{3}+2i$

(c) $6+10i$

(d) $-1+5i$

(e) $\frac{1}{2}i$

(f) $\frac{9}{17}+\frac{2}{17}i$

3.

(a) $2\left[\cos\left(\frac{\pi}{3}+2k_1\pi\right)+i\cdot\sin\left(\frac{\pi}{3}+2k_1\pi\right)\right] \quad k_1\in Z$

(b) $2\left[\cos\left(\frac{\pi}{4}+2k_2\pi\right)+i\cdot\sin\left(\frac{\pi}{4}+2k_2\pi\right)\right] \quad k_2\in Z$

(c) $4\left[\cos\left(\frac{3\pi}{2}+2k_3\pi\right)+i\cdot\sin\left(\frac{3\pi}{2}+2k_3\pi\right)\right]\quad k_3\in Z$

(d) $8\left[\cos(2k_4\pi)+i\cdot\sin(2k_4\pi)\right]\quad k_4\in Z$

(e) $\sqrt{2^3}\left[\cos\left(\frac{11\pi}{6}+2k_5\pi\right)+i\cdot\sin\left(\frac{11\pi}{6}+2k_5\pi\right)\right]\quad k_5\in Z$

(f)
$$\sqrt{10}\left[\cos\left(\arccos(3/\sqrt{10})+2k_6\pi\right)+i\cdot\sin\left(\arcsin(1/\sqrt{10})+2k_6\pi\right)\right]\approx$$
$$\approx\sqrt{10}\left[\cos\left(\frac{\pi}{10}+2k_6\pi\right)+i\cdot\sin\left(\frac{\pi}{10}+2k_6\pi\right)\right]\quad k_6\in Z$$

4.

(a) $\sqrt{2}\left[\cos\left(\frac{\frac{\pi}{2}+2k\pi}{2}\right)+i\cdot\sin\left(\frac{\frac{\pi}{2}+2k\pi}{2}\right)\right]\quad k=0,\ 1\ \rightarrow$ a gyökök: $1+i\,;\ -1-i$

(b) $1\,;\ -\frac{1}{2}+\frac{\sqrt{3}}{2}i\,;\ -\frac{1}{2}-\frac{\sqrt{3}}{2}i$

(c) $\sqrt{2}+i\sqrt{2}\,;\ \sqrt{2}-i\sqrt{2}\,;\ -\sqrt{2}+i\sqrt{2}\,;\ -\sqrt{2}-i\sqrt{2}$

(d)
$$2\left[\cos\left(\frac{\frac{3\pi}{4}+2k\pi}{3}\right)+i\cdot\sin\left(\frac{\frac{3\pi}{4}+2k\pi}{3}\right)\right]k=0,\ 1,\ 2\rightarrow\sqrt{2}+i\sqrt{2}\,;\ -1.932+i\,0.518\,;\ 0.518-i\,1.932$$

5.

(a) -8

(b) -4

(c) 16

6.

(a) $z_{1,2}=3\pm 2i$

(b) $z_{1,2,3}=\sqrt[3]{1+i} \;\rightarrow\; z_1=1.084+0.291\cdot i\,;\;\; z_2=-\frac{1}{\sqrt[3]{2}}+\frac{1}{\sqrt[3]{2}}\cdot i\,;\;\; z_3=-0.291-1.084\cdot i$

(c) $z=\frac{3}{2}-2\,i$

(d) $z=0$

(e) $z=-1\,;0\,;1$ **kielégítik**

(f) $z_{1,2,3,4}=\sqrt{1\pm\sqrt{3}\,i} \;\rightarrow\; z_1=\sqrt{\frac{3}{2}}+\frac{1}{\sqrt{2}}\,i\,;\;\; z_2=-\sqrt{\frac{3}{2}}-\frac{1}{\sqrt{2}}\,i\,;\;\; z_3=\sqrt{\frac{3}{2}}-\frac{1}{\sqrt{2}}\,i\,;\;\; z_4=-\sqrt{\frac{3}{2}}+\frac{1}{\sqrt{2}}\,i$

(g) $z_{1,2,3,4,5,6}=\sqrt[3]{1\pm i} \;\rightarrow\; z_{1,4}=1.084\pm 0.291\cdot i\,;\;\; z_{2,5}=-\frac{1}{\sqrt[3]{2}}\pm\frac{1}{\sqrt[3]{2}}\cdot i\,;\;\; z_{3,6}=-0.291\mp 1.084\cdot i$