Formula sheet
Mathematics A1a

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}, \quad \binom{n}{k} = \binom{n}{n-k}, \quad \binom{n}{0} = 1,$$

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{n-1}ab^{n-1} + \binom{n}{n}b^n$$

$$\sin^2 x + \cos^2 x = 1$$
$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$
$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$
$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \cdot \tan y}$$
$$\sin 2x = 2 \sin x \cos x$$
$$\cos 2x = \cos^2 x - \sin^2 x$$
$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$
$$\sin^2 x = \frac{1 - \cos 2x}{2}, \quad \cos^2 x = \frac{1 + \cos 2x}{2}$$
$$\sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}$$
$$\sin x - \sin y = 2 \cos \frac{x+y}{2} \sin \frac{x-y}{2}$$
$$\cos x + \cos y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}$$
$$\cos x - \cos y = -2 \sin \frac{x+y}{2} \sin \frac{x-y}{2}$$
$$\sin x \cos y = \frac{1}{2}[\sin(x+y) + \sin(x-y)]$$
$$\cos x \cos y = \frac{1}{2}[\cos(x+y) + \cos(x-y)]$$
$$\sin x \sin y = -\frac{1}{2}[\cos(x+y) - \cos(x-y)]$$
$$\sinh x = \frac{e^x - e^{-x}}{2}, \quad \cosh x = \frac{e^x + e^{-x}}{2}$$
$$\cosh^2 x - \sinh^2 x = 1$$
$$\sinh 2x = 2 \sinh x \cosh x$$
$$\cosh 2x = \cosh^2 x + \sinh^2 x$$
$$\cosh^2 x = \frac{\cosh 2x + 1}{2}, \quad \sinh^2 x = \frac{\cosh 2x - 1}{2}$$

Rules of differentiation:

$$(cu)' = cu' \quad (c \text{ konstans})$$
$$(u+v)' = u' + v'$$
$$(uv)' = u'v + uv'$$
$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$
$$\frac{df}{dx} = \frac{df}{dg}\frac{dg}{dx}, \text{ if } f = f(g(x))$$

Rules of indefinite integrals:

$$\int cf\,dx = c\int f\,dx \quad (c \text{ konstans})$$
$$\int (f+g)dx = \int f\,dx + \int g\,dx$$
$$\int f(ax+b)dx = \frac{1}{a}F(ax+b) + c,$$
where F is an antiderivative of f
$$\int f(g(x))g'(x)dx = F(g(x)) + c,$$
$$\int f^{\mu} f' dx = \frac{f^{\mu+1}}{\mu+1} + c, \text{ if } \mu \neq -1$$
$$\int \frac{f'}{f}dx = \ln|f| + c$$
$$\int uv'\,dx = uv - \int u'v\,dx$$

$$\left(x^n\right)' = nx^{n-1}$$
$$\left(e^x\right)' = e^x$$
$$\left(a^x\right)' = a^x \ln a$$
$$(\sin x)' = \cos x$$
$$(\cos x)' = -\sin x$$
$$(\tan x)' = \frac{1}{\cos^2 x}$$
$$(\cot x)' = -\frac{1}{\sin^2 x}$$
$$(\sinh x)' = \cosh x$$
$$(\cosh x)' = \sinh x$$
$$(\ln x)' = \frac{1}{x}$$
$$(\log_a x)' = \frac{1}{x \ln a}$$

$$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$$
$$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$$
$$(\arctan x)' = \frac{1}{1+x^2}$$
$$(\operatorname{arc}\cot x)' = -\frac{1}{1+x^2}$$
$$(\operatorname{ar}\sinh x)' = \frac{1}{\sqrt{1+x^2}}$$
$$(\operatorname{ar}\cosh x)' = \frac{1}{\sqrt{x^2-1}}$$
$$(\operatorname{ar}\tanh x)' = \frac{1}{1-x^2}$$
$$(\operatorname{ar}\coth x)' = \frac{1}{1-x^2}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c \quad (n \neq -1)$$
$$\int \frac{1}{x} dx = \ln|x| + c$$
$$\int a^x dx = \frac{a^x}{\ln a} + c$$
$$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$$
$$\int \sin x dx = -\cos x + c$$
$$\int \cos x dx = \sin x + c$$
$$\int \tan x dx = -\ln|\cos x| + c$$
$$\int \cot x dx = \ln|\sin x| + c$$
$$\int \frac{1}{\cos^2 x} dx = \tan x + c$$
$$\int \frac{1}{\sin^2 x} dx = -\cot x + c$$

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\arctan\frac{x}{a} + c$$
$$\int \frac{dx}{a^2-x^2} = \begin{cases} \frac{1}{a}\operatorname{ar}\ \tanh\frac{x}{a} + c, \text{ ha } \left|\frac{x}{a}\right| < 1 \\ \frac{1}{a}\operatorname{ar}\ \coth\frac{x}{a} + c, \text{ ha } \left|\frac{x}{a}\right| > 1 \end{cases}$$
$$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + c$$
$$\int \frac{dx}{\sqrt{x^2+a^2}} = \operatorname{ar}\ \sinh\frac{x}{a} + c$$
$$\int \frac{dx}{\sqrt{x^2-a^2}} = \operatorname{ar}\ \cosh\frac{x}{a} + c$$

Taylor's polynomial: $T_n(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + ... + \frac{f^{(n)}(a)}{n!}(x-a)^n$

Curvature: $G = \frac{y''}{(1+y'^2)^{3/2}}$

Substitutions:

$R(e^x)$	$e^x = t$
$R(\sqrt{ax+b})$	$\sqrt{ax+b} = t$
$R\left(\frac{\sqrt{ax+b}}{\sqrt{cx+d}}\right)$	$\frac{\sqrt{ax+b}}{\sqrt{cx+d}} = t$
$R(\sin x, \cos x)$	$\sin x = t, \cos x = t, \operatorname{tg} x = t, \operatorname{tg}\frac{x}{2} = t$
$R(x, \sqrt{a^2 - x^2})$	$x = a\sin t, x = a\cos t$
$R(x, \sqrt{a^2 + x^2})$	$x = a \operatorname{sh} t$
$R(x, \sqrt{x^2 - a^2})$	$x = a \operatorname{ch} t$

Applications of definite integrals

1. Area $T = \int_a^b f(x)dx$ $\quad T = \int_{t_1}^{t_2} y\dot{x}dt$ $\quad T = \frac{1}{2}\int_{\varphi_1}^{\varphi_2} r^2(\varphi)d\varphi$

2. Arc length $s = \int_a^b \sqrt{1+f'^2(x)}dx$ $\quad s = \int_{t_1}^{t_2} \sqrt{\dot{x}^2 + \dot{y}^2}dt$ $\quad s = \int_{\varphi_1}^{\varphi_2} \sqrt{r^2 + \dot{r}^2}d\varphi$

3. Volume of solids of rotation $V = \pi\int_a^b f^2(x)dx$ $\quad V = \pi\int_{t_1}^{t_2} y^2\dot{x}dt$

4. Surface area of solids of rotation $F = 2\pi\int_a^b f(x)\sqrt{1+f'^2(x)}dx$, $F = 2\pi\int_{t_1}^{t_2} y\sqrt{\dot{x}^2 + \dot{y}^2}dt$

5. Moments: $M_x = \frac{1}{2}\int_a^b f^2(x)dx$ $\quad M_y = \int_a^b xf(x)dx$

6. Centroid $(x_s; y_s)$: $x_s = \frac{M_y}{T}$ $\quad y_s = \frac{M_x}{T}$